

## **ECONOMIC INTERDEPENDENCE WITHIN THE CHICAGO METROPOLITAN AREA: A MIYAZAWA ANALYSIS\***

**Geoffrey J.D. Hewings**

*Regional Economics Applications Laboratory, University of Illinois, Urbana, IL 61801, U.S.A. E-mail: hewings@uiuc.edu*

**Yasuhide Okuyama**

*Department of Planning, State University of New York at Buffalo, Buffalo, NY 14214, U.S.A. E-mail: yokuyama@ap.buffalo.edu*

**Michael Sonis**

*Regional Economics Applications Laboratory, University of Illinois, Urbana, IL 61801, U.S.A. and Bar Ilan University, Israel. E-mail: sonism@mail.biu.ac.il*

**ABSTRACT.** The present study explores the nature and strength of economic interdependence between inner-city communities and suburbs within the Chicago metropolitan area. Employing Miyazawa's extended input-output framework, a multiregional model is used to investigate the interdependence of income formation and output generation. The metropolitan area is divided into four regions and particular attention is directed to predominantly minority areas on the south and west sides of the city of Chicago. The region-to-region impacts of trade flows and their associated multipliers proved to be far less important in determining the strength of interregional interdependence in contrast to income flows derived from journey-to-work movements. The interrelational income multiplier revealed considerable interdependence between regions although the strength of this interdependence was asymmetric.

### 1. INTRODUCTION

Policies targeting the economic development of inner-city communities within metropolitan areas cover a spectrum involving mixes of active intervention that range from tax increment financing, empowerment zones and other similar schemes to more modest attempts to target key aspects of infrastructure;

---

\*This project was made possible with generous support from the John D. and Catherine T. MacArthur Foundation. Professor David Boyce (University of Illinois at Chicago) and his research assistants provided significant contributions in preparing the journey-to-work data for use in the analysis. The Project Steering Committee appointed by the Chicago United and the enthusiastic support of Carolyn Nordstrom and her staff provided invaluable guidance in helping shape the analysis. We thank the many providers of comments received at conference presentations as well as the comments of the referees. The views expressed in this paper are those of the authors and do not represent official positions of the Federal Reserve Bank of Chicago.

Received August 1999; revised June 2000; accepted August 2000.

the contributions, successes, and failures in the United States are well documented in the literature (see, for example, Goldsmith and Blakey, 1992; Harrison, 1974). Although the insights derived from the application of models providing explicit consideration of sectoral and spatial interdependence feature prominently in national- and regional-scale policy analysis, there have been limited attempts to secure these perspectives in the context of interactions between areas within metropolitan economies. Issues of scale, a dearth of appropriate data, and an unclear sense of their use may be offered as explanation, and perhaps a sense that communities within metropolitan regions were characterized more by social, housing, or political characteristics than ones based on economic transactions. Some earlier attempts to analyze community-level economies were made using economic base multipliers (for example, Vietorisz and Harrison, 1970; Mellor, 1972; Schaffer, 1973). The theoretical limitations of the economic base model and the real difficulties posed by data collection at smaller geographic scales resulted in relatively few new attempts along these lines, with the possible exception of Cole (1994, 1999), whose Community Accounting Matrix (CAM) based on the Social Accounting Matrix (SAM) framework was constructed "as a sketch pad to explore a characteristic structure of activities and relationships" (1999, p. 269) between an inner-city community (East Side) in Buffalo and the rest of the metropolitan area (Erie county).

In many metropolitan regions, conflicts between central cities and suburbs have been waged on the premise that neither area needs the other and, for the most part, these assertions have gone unchallenged with little if any sound economic analysis to provide a foundation for their support or refutation. In this climate, inner-city development is often seen as a zero-sum game, providing little demonstrable benefit to parts of the metropolitan region outside the targeted areas and commanding public resources with high opportunity costs that might be more effectively directed to other parts of the region. Again, little formal analysis has been conducted to examine the nature, strength, and type of any economic spillover and thus challenge the veracity of these assertions. Yet, in general, if there are gains from trade and interdependence between nations or between regions within a nation, should there not be some expectation of similar findings within a metropolitan region? It is from this premise that the current exercise is launched. An understanding and appreciation of the magnitudes of the economic relationships and economic interdependence between inner-city communities and the rest of metropolitan area may prove to be strategic information in the analysis of the region-wide impact of inner-city development. Unlike trade between nations, this interdependence depends not only on the movement of goods and services but also on the movement of labor, that is, commuting and the associated income flows. To illustrate the complex interdependencies within a metropolitan area a multiregional input-output model was constructed using Miyazawa's (1976) extended framework to explore the benefits to all parts of the region from economic initiatives generated in one area and

to provide the basis for the notion that the gains from trade, promoted at the international level, can be also realized within a metropolitan area. To analyze the economic interdependence between inner-city communities and suburbs, the Chicago metropolitan area is divided into four regions with particular attention directed to predominantly minority areas on the south and west parts of the City of Chicago.

In the next section, the theoretical framework of the model is introduced. Section 3 describes the data and the structure of the model. In Section 4 the results of the model are presented and analyzed. Section 5 offers some policy interpretations and Section 6 concludes this paper with some suggestions for extensions of this work.

## 2. MIYAZAWA'S FRAMEWORK

This section draws on Miyazawa's (1976) extended input-output framework, see Hewings et al. (1999) for a collection of papers describing the methodology in more detail and illustrating applications of this framework. In particular, it focuses on the estimation of (1) the interrelational income multiplier and (2) the internal and external multipliers for the evaluation of the linkages and interdependence between regions.

### *Interrelational Income Multiplier*

Miyazawa's (1976) concept of the interrelational income multiplier was designed to analyze the structure of income distribution by endogenizing consumption demands in the standard Leontief model. In an interregional context, the inclusion of the income formation process has clear advantages for linking the location of production (or wage earning) and the location of consumption. In some sense, Miyazawa's system may be considered the most parsimonious in terms of the way it extends the familiar input-output formulation.<sup>1</sup> Miyazawa considered the system shown in Equation (1)

$$(1) \quad \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{V} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} + \begin{pmatrix} \mathbf{f} \\ \mathbf{g} \end{pmatrix}$$

where  $\mathbf{X}$  is a vector of output,  $\mathbf{Y}$  is a vector of total income for some  $r$ -fold division of income groups,  $\mathbf{A}$  is a block matrix of direct input coefficients,  $\mathbf{V}$  is a matrix of value-added ratios for  $r$ -fold income groups (or regions in an interregional context),  $\mathbf{C}$  is a corresponding matrix of consumption coefficients,  $\mathbf{f}$  is a vector

<sup>1</sup>Pyatt (2001) has recently drawn attention to the formal distinction between a Miyazawa system and its close relation the social accounting matrix. He proposes that the income multipliers associated with Miyazawa be referred to as factorial income multipliers because they do not include other institutional contributions to income (i.e., nonwage and salary income such as dividends, transfers, and pensions). Thus, the SAM income is defined more extensively and the associated multipliers should be distinguished as institutional income multipliers. However, Equation (2) does contain exogenous income and total income formation would be  $\mathbf{Y} = \mathbf{KVf} + \mathbf{Kg}$ ; but in the definition of the interrelational income multipliers  $\mathbf{K}$  only endogenously determined income is included.

of final demands except households consumption, and  $\mathbf{g}$  is a vector of exogenous income for  $r$ -fold income groups. Solving this system yields

$$(2) \quad \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \end{pmatrix} = \left[ \begin{array}{c|c} \mathbf{B}(\mathbf{I} + \mathbf{CKVB}) & \mathbf{BCK} \\ \hline \mathbf{KVB} & \mathbf{K} \end{array} \right] \begin{pmatrix} \mathbf{f} \\ \mathbf{g} \end{pmatrix}$$

where  $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$  is the Leontief inverse matrix,  $\mathbf{BC}$  is a matrix of production induced by endogenous consumption,  $\mathbf{VB}$  is a matrix of endogenous income earned from production,  $\mathbf{L} = \mathbf{VBC}$  is a matrix of expenditures from endogenous income, and  $\mathbf{K} = (\mathbf{I} - \mathbf{L})^{-1}$  is a matrix of the Miyazawa interrelational income multipliers. In an interregional formulation, this framework is able to provide a clear picture of interdependence between regions, in terms of income formation and output generation. The interrelational income multipliers in Equation (2),  $\mathbf{K} = (\mathbf{I} - \mathbf{L})^{-1}$ , indicate how the increase in income formulation in one region can generate income in the remaining regions.

#### *Internal and External Multipliers*

Miyazawa's "internal" and "external" multipliers were derived to partition the standard Leontief inverse to enable analysts to separate demands into those generated internally (within the region) from those originating in the remaining regions of the system. Consider a two-region system represented in the following block structure

$$\mathbf{A} = \left( \begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array} \right)$$

where  $\mathbf{A}_{11}$  and  $\mathbf{A}_{22}$  are the intraregional matrices of direct inputs within the first and second region, and  $\mathbf{A}_{12}$  and  $\mathbf{A}_{21}$  are the interregional matrices representing direct input connections between Regions 1 and 2. The standard Leontief inverse then takes the form

$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1} = \left( \begin{array}{c|c} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \hline \mathbf{B}_{21} & \mathbf{B}_{22} \end{array} \right)$$

Miyazawa claimed that this standard Leontief inverse displays only the total system-wide effects of the economy, and introduced a decomposition of the Leontief inverse in order to separate the components of the two regions as follows

$$\begin{cases} \mathbf{B}_1 = (\mathbf{I} - \mathbf{A}_{11})^{-1} \\ \mathbf{B}_2 = (\mathbf{I} - \mathbf{A}_{22})^{-1} \end{cases}$$

where  $\mathbf{B}_1$  and  $\mathbf{B}_2$  are the Miyazawa internal matrix multiplier for the Regions 1 and 2, respectively. Using these internal matrix multipliers, interregional propagation activities will be shown as four rectangular sub-matrix multipliers

$$\begin{cases} \mathbf{P}_1 = \mathbf{A}_{21}\mathbf{B}_1 \\ \mathbf{P}_2 = \mathbf{B}_1\mathbf{A}_{12} \end{cases}$$

and

$$\begin{cases} \mathbf{S}_1 = \mathbf{A}_{12}\mathbf{B}_2 \\ \mathbf{S}_2 = \mathbf{B}_2\mathbf{A}_{21} \end{cases}$$

where:

$\mathbf{P}_1$  is the matrix multiplier indicating input from Region 1 to Region 2 induced by internal propagation in Region 1;

$\mathbf{P}_2$  is the matrix multiplier for internal propagation in Region 1 induced by transactions from Regions 1 to 2;

$\mathbf{S}_1$  is the matrix multiplier of input from Regions 1 to 2 induced by internal propagation in Region 2; and

$\mathbf{S}_2$  is the matrix multiplier for internal propagation in Region 2 induced by transactions from Regions 2 to 1.

Employing these sub-matrix multipliers, the external matrix multipliers for the regions can be derived as follows

$$\Delta_{11} = (\mathbf{I} - \mathbf{P}_2\mathbf{S}_2)^{-1} = (\mathbf{I} - \mathbf{B}_1\mathbf{A}_{12}\mathbf{B}_2\mathbf{A}_{21})^{-1}$$

$$\Delta_{22} = (\mathbf{I} - \mathbf{S}_2\mathbf{P}_2)^{-1} = (\mathbf{I} - \mathbf{B}_2\mathbf{A}_{21}\mathbf{B}_1\mathbf{A}_{12})^{-1}$$

where  $\Delta_{11}$  and  $\Delta_{22}$  are the Miyazawa external matrix multipliers, indicating the external propagation activities, for Regions 1 and 2, respectively. Using an explicit hierarchical order among the regions with this matrix decomposition technique, Sonis and Hewings (1993) identified the following multiplicative structure of Leontief inverse for Miyazawa partitioned multipliers

$$(3) \quad (\mathbf{I} - \mathbf{A})^{-1} = \begin{pmatrix} \Delta_{11} & 0 \\ 0 & \Delta_{22} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{B}_1\mathbf{A}_{12} \\ \mathbf{B}_2\mathbf{A}_{21} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{B}_1 & 0 \\ 0 & \mathbf{B}_2 \end{pmatrix}$$

In the formulation shown in Equation (3), the Miyazawa internal and external, intraregional multipliers are separated from the interregional effects as they are presented in the standard Leontief inverse.

In the following section, the model is constructed based on the Chicago Region Input-Output table.

### 3. THE MODEL

The Chicago metropolitan area (including six counties: Cook, DuPage, Kane, Lake, McHenry, and Will) is divided into four smaller regions: Region 1—Loop and North Side; Region 2—South Side; Region 3—West Side; and Region 4—

Suburbs. Regions 1, 2, and 3 consist of the City of Chicago (see Figure 1 for location; and see Table 1 for size) to investigate the spatial interaction between the regions. Regions 2 and 3 are areas with significant concentrations of African-American and Hispanic populations, respectively. A four-region input-output model was constructed based on the Chicago Region Econometric Input-Output Model, and is extended to Miyazawa's framework for the analysis of interdependence among these regions.

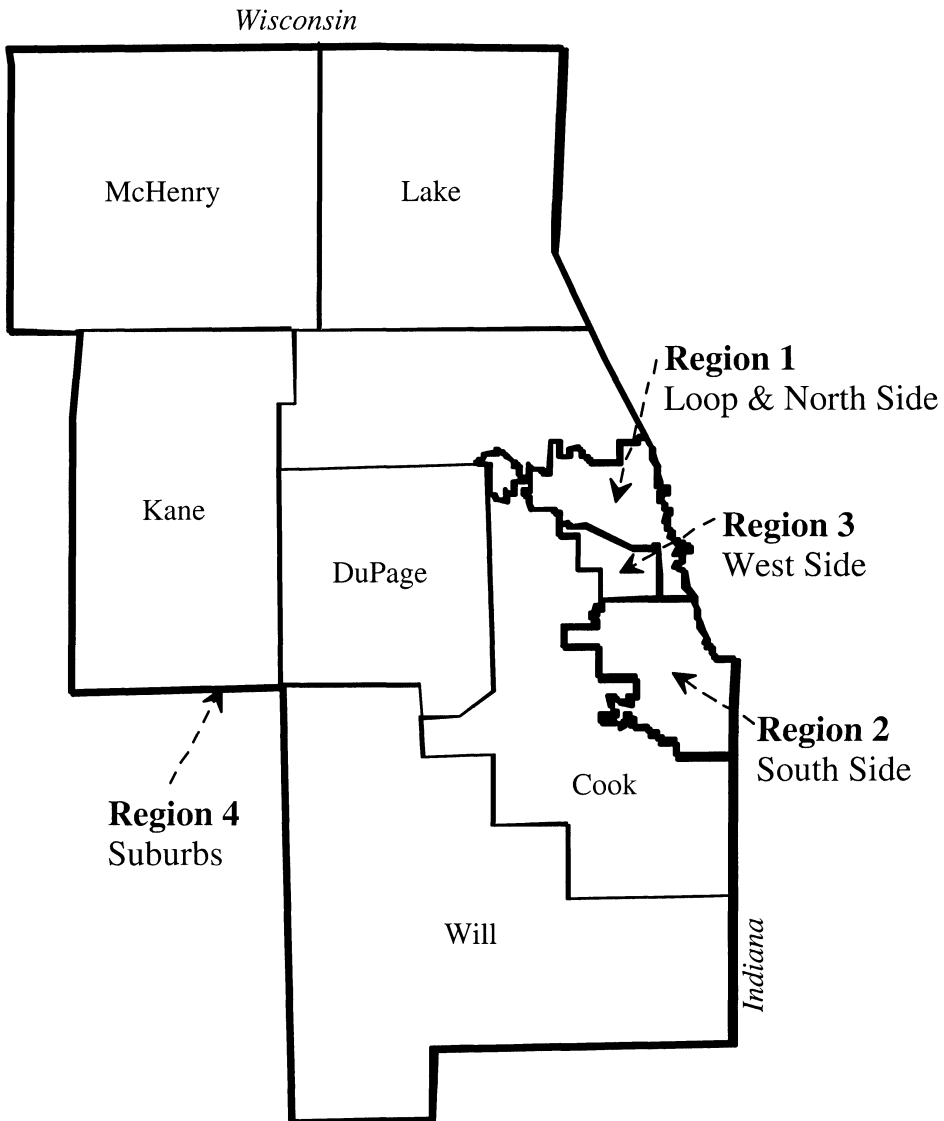


FIGURE 1: Location of the Four Regions.

TABLE 1: Size of Four Regions: Employment by Region Based on Place of Work, 1995

Region	Employment	Percentage
Region 1	769,829	24.20
Region 2	184,131	5.79
Region 3	123,874	3.89
Region 4	2,102,881	66.11
Total	3,180,715	100.00

### *Estimation of Trade Coefficients*

The Chicago four-region input-output table is constructed by modifying the 1992 regional input-output table for the Chicago metropolitan area, derived from the Chicago Region Econometric Input-output Model (CREIM) with 53 industrial sectors.<sup>2</sup> Using the employment data for the four regions by the Illinois Department of Employment Security (1996), the regional input-output table was disaggregated geographically into a four-region multiregional model, using location quotient adjustment and biproportional balancing. A typical location quotient of sector  $i$  in region  $p$  can be written as

$$LQ_i^p = \left[ \frac{e_i^p}{e^p} \right] \bigg/ \left[ \frac{E_i}{E} \right]$$

where  $e_i^p$  is the employment of sector  $i$  in region  $p$ ,  $e^p$  is the total employment in region  $p$ ,  $E_i$  is the employment of sector  $i$  in benchmark (usually state or nation), and  $E$  is the total employment of benchmark. In this case, the benchmark is set as the total employment of the Chicago Metropolitan region (equivalent to the total of the four regions). Then, the export shares by sector for the four regions are derived as follows

$$ex_i^p = \left( 1 - \frac{1}{LQ_i^p} \right)$$

where  $ex_i^p$  is the export share of sector  $i$  in region  $p$ .

The export shares indicate the export portion of employment, and also of output assuming that the relationship between output and employment is constant for any sector no matter where located in the metropolitan region; hence, these export shares can be seen as row trade coefficients in the multi-regional input-output literature (Polenske, 1970). Dividing this export share into three receiving regions using the total employment of each region and the distance factor, the row trade coefficient  $r_i^{pq}$  for sector  $i$  from regions  $p$  to  $q$ , is

<sup>2</sup>More details of the model are found in Israilevich et al. (1997).

derived. By definition, the row trade coefficients add up to unity over the purchasing regions as follows

$$\sum_q r_i^{pq} = 1$$

Based on Polenske (1970), the row coefficient model is given by

$$(4) \quad \sum_p r_i^{pq} x_i^p = \sum_j a_{ij}^q x_j^q + y_i^q$$

In matrix form, Equation (4) can be transformed as

$$\mathbf{R}'\Delta\mathbf{X} = \hat{\mathbf{A}}\Delta\mathbf{X} + \Delta\mathbf{Y}$$

or

$$\Delta\mathbf{X} = (\mathbf{R}' - \hat{\mathbf{A}})^{-1} \Delta\mathbf{Y}$$

where  $\mathbf{R}$  is a square matrix filled with diagonal matrices of  $r_i^{pq}$ . However, due to the mathematical properties of this formulation, the row coefficient model inevitably generates negative estimation of elements in  $(\mathbf{R}' - \hat{\mathbf{A}})^{-1}$  (Bon, 1975, 1984; Toyomane, 1988).

With this set of trade coefficients, the row trade coefficients are transformed to column trade coefficient imposing the following constraints

$$\sum_p c_i^{pq} = 1$$

where  $c_i^{pq}$  is the column trade coefficient of sector  $i$  from regions  $p$  to  $q$ . This transformation is carried out as follows

$$c_i^{pq} = \frac{r_i^{pq} e^p}{\sum_p r_i^{pq} e^p}$$

where  $e^p$  is the total employment of region  $p$  (for more details see Appendix).

#### *Construction of Four-Region Multiregional Model*

Using the column trade coefficients calculated above, a four-region multi-regional model can be derived as follows

$$x_i^p = \sum_q \sum_j c_i^{pq} a_{ij}^q x_j^q + \sum_q c_i^{pq} y_i^q$$

In matrix form, the column coefficient model can be transformed as



$$\Delta \mathbf{X} = \mathbf{C}(\hat{\mathbf{A}}\Delta \mathbf{X} + \Delta \mathbf{Y})$$

or

$$\Delta \mathbf{X} = (\mathbf{I} - \mathbf{C}\hat{\mathbf{A}})^{-1} \mathbf{C}\Delta \mathbf{Y}$$

and

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & 0 & 0 & 0 \\ 0 & \mathbf{A} & 0 & 0 \\ 0 & 0 & \mathbf{A} & 0 \\ 0 & 0 & 0 & \mathbf{A} \end{bmatrix}$$

where  $\mathbf{A}$  is the matrix of direct input coefficients in the Chicago metropolitan region and is obtained from CREIM. Using this  $\hat{\mathbf{A}}$ , the interregional Leontief inverse of this four-region input-output system is derived by calculating  $(\mathbf{I} - \mathbf{C}\hat{\mathbf{A}})^{-1}$ .

This column coefficient model is clearly a multiregional one that assumes non-competitive trade based on the trade pool theory.<sup>3</sup> Although this noncompetitive assumption may be tenable for multiregional modeling in a larger geographical aggregation (for example, in international and interregional levels), it might not be plausible for intrametropolitan trade. However, the evidence of the hollowing-out process in the Chicago metropolitan area (Hewings et al., 1998) indicates that industries, especially manufacturing sectors, have been engaging in greater interregional trade in volume with the industries outside the Chicago area than with the industries within the Chicago area. Thus, although the noncompetitive assumption is applied here, the results from the model should be analyzed carefully.

#### *Extended Model using Miyazawa's Framework*

To extend the four-region input-output table to the Miyazawa's framework, value-added coefficients, and consumption coefficients in each region must be determined. The data for these coefficients are derived from the 1990 CATS Household Travel Survey by the Chicago Area Transportation Study (1994). From these data, two sets of trip data for the Chicago metropolitan region were extracted: journey-to-work and journey-to-shop trips. Value-added coefficients are derived using the journey-to-work data of eight income groups adjusted by county income data from Regional Economic Information System (REIS) by the U.S. Department of Commerce. Consumption coefficients are calculated based on the CATS journey-to-shop data and consumption data from CREIM. Using the multiregional transaction matrix derived in the previous

<sup>3</sup>Trade pool theory was discussed by Isard and Bramhall (1960), and later proposed in a gravity formulation by Leontief and Strout (1963). The column trade coefficient model used here was originally introduced by Chenery (1953) in a two-region study of Italy, and later extended to a three-region model by Moses (1955).

section, the addition of these value-added and consumption matrices to the model now completes Miyazawa's extended input-output formulation shown in Equation (1).

#### 4. ANALYSIS OF ECONOMIC INTERDEPENDENCE

The results from the four-region input-output model are presented and analyzed in this section. First, attention is directed to the trade flows of goods and services between the four regions, together with the interpretation offered by Miyazawa's distinction between internal and external multipliers. Thereafter, the focus shifts to flows of labor (commuting) and associated income flows within the metropolitan area. Finally, combining these two observations, Miyazawa's interrelational income multipliers are presented and analyzed.

##### *Trade Flows: Aggregate Analysis*

The estimated four-region input-output model provides the flows of goods and services between sectors within the Chicago metropolitan area; the dimension of the matrix becomes 212 by 212 because the model has 53 sectors for each region. Attempting to show this matrix in a convenient fashion is a daunting problem. Instead, two alternative aggregations are made—first at the one-sector level, and then the analysis is conducted at the nine-sector level to illustrate Miyazawa's distinction between internal and external multipliers.

Table 2 shows the aggregate trade flows of intermediate goods between the four regions. Not surprisingly, the entries on the diagonal reveal that trade within each region is significantly larger than any interregional trade. This tendency may appear rather striking given the geographical closeness of these regions; however, the metropolitan area as a whole has experienced a significant hollowing out of the past two decades, whereby local intermediate transactions have declined although total production has increased (Hewings et al., 1998). Figure 2 illustrates the direction and volume (thickness of arrow) of the net exports between the regions. Region 1, containing the central business district (CBD) and the north side of the city, has a positive trade relationship with all of the remaining three regions. Region 4, the suburbs, also has positive trade with Regions 2 and 3. However, again, these trades are substantially smaller compared to the intraregional trade.

TABLE 2: Aggregate Trade Flows (Intermediate Goods) between the Regions [\$ 1992 millions]

From/To	Region 1	Region 2	Region 3	Region 4	Total
Region 1	19,343	401	293	2,886	22,924
Region 2	137	3,770	67	400	4,375
Region 3	74	110	3,072	246	3,501
Region 4	41,516	786	395	60,265	62,962
Total	21,071	5,067	3,827	63,798	93,762

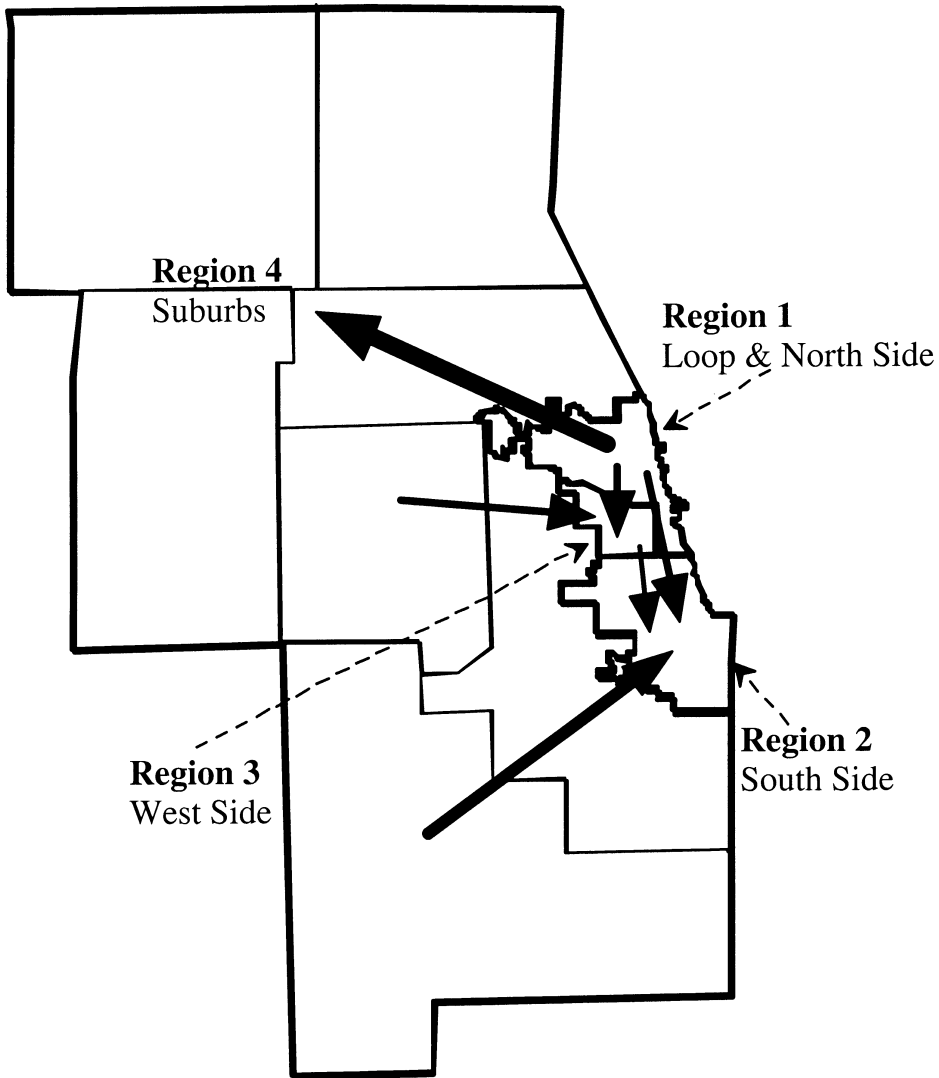


FIGURE 2: Net Exports Between the Regions.

As described in Section 2, Miyazawa's internal and external multipliers present the degree of economic interaction between two regions through internal and external propagation activities, respectively. Miyazawa's formulation is based on a two-region system so the internal and external multipliers are derived as one region versus the rest of Chicago: with the second region

comprising an aggregation of the remaining three regions.<sup>4</sup> Tables 3 through 6 show the internal and external multipliers for each region versus the rest of Chicago in a 9-sector version. On average, Region 4 has the largest internal multiplier (1.42), followed by Regions 1, 3, and 2. This indicates that Region 4 is more self-contained than other regions. This tendency continues in the total multiplier effect, the product of internal and external multipliers in a region:

TABLE 3: Internal and External Multipliers of Two-Region Model (Region 1)

	Region 1				Rest of Chicago			
	Internal Row	Column	External Row	Column	Internal Row	Column	External Row	Column
Agriculture	1.0227	1.2901	1.0001	1.0007	1.0552	1.3436	1.0001	1.0004
Construction	1.1783	1.4579	1.0001	1.0017	1.2389	1.5431	1.0008	1.0005
Manufacturing (Non-Durable)	1.4472	1.3079	1.0004	1.0009	1.6333	1.4665	1.0014	1.0006
Manufacturing (Durable)	1.4465	1.3852	1.0002	1.0012	1.6687	1.4053	1.0016	1.0004
TCU	1.4154	1.3767	1.0013	1.0010	1.4352	1.4056	1.0004	1.0009
Trades	1.2107	1.3138	1.0001	1.0005	1.3153	1.3229	1.0006	1.0009
FIRE	1.6476	1.4547	1.0025	1.0003	1.4870	1.4225	1.0002	1.0013
Services	1.9503	1.4015	1.0036	1.0007	1.9917	1.4446	1.0014	1.0009
Governments	1.0331	1.3639	1.0001	1.0014	1.0305	1.5018	1.0000	1.0007
Average	1.3724	1.3724	1.0009	1.0009	1.4284	1.4284	1.0007	1.0007

TABLE 4: Internal and External Multipliers of Two-Region Model (Region 2)

	Region 2				Rest of Chicago			
	Internal Row	Column	External Row	Column	Internal Row	Column	External Row	Column
Agriculture	1.0106	1.2414	1.0000	1.0002	1.0538	1.3580	1.0001	1.0003
Construction	1.1774	1.3976	1.0000	1.0005	1.2404	1.5644	1.0000	1.0004
Manufacturing (Non-Durable)	1.3867	1.2676	1.0016	1.0005	1.6184	1.4725	1.0007	1.0007
Manufacturing (Durable)	1.4696	1.3287	1.0009	1.0005	1.6475	1.4185	1.0007	1.0007
TCU	1.2985	1.2566	1.0001	1.0003	1.4793	1.4431	1.0002	1.0001
Trades	1.2252	1.2467	1.0001	1.0002	1.3080	1.3554	1.0002	1.0001
FIRE	1.4490	1.3440	1.0000	1.0002	1.6191	1.4853	1.0002	1.0000
Services	1.7301	1.3375	1.0003	1.0003	2.0782	1.4728	1.0004	1.0003
Governments	1.0305	1.3574	1.0000	1.0004	1.0324	1.5072	1.0000	1.0001
Average	1.3086	1.3086	1.0003	1.0003	1.4530	1.4530	1.0003	1.0003

<sup>4</sup>Miyazawa's external multipliers can be derived for an interregional system with more than two regions; however, the calculation of these multipliers requires a priori specification of hierarchy (order) of propagation route. And, with a  $n$ -region system it produces  $(n + 1)!/2$  combinations of propagation routes.

TABLE 5: Internal and External Multipliers of Two-Region Model (Region 3)

	Region 3		External		Rest of Chicago		External	
	Internal Row	Column	Row	Column	Internal Row	Column	Row	Column
Agriculture	1.0149	1.1652	1.0000	1.0003	1.0536	1.3470	1.0000	1.0010
Construction	1.1662	1.4299	1.0000	1.0004	1.2404	1.5641	1.0001	1.0003
Manufacturing (Non-Durable)	1.3959	1.4047	1.0006	1.0003	1.6313	1.4621	1.0005	1.0004
Manufacturing (Durable)	1.5241	1.4260	1.0010	1.0003	1.6409	1.4176	1.0006	1.0004
TCU	1.2880	1.2651	1.0001	1.0002	1.4789	1.4424	1.0002	1.0000
Trades	1.2354	1.2459	1.0000	1.0001	1.3082	1.3564	1.0002	1.0000
FIRE	1.3936	1.4528	1.0000	1.0001	1.6193	1.4836	1.0003	1.0000
Services	2.0622	1.3343	1.0005	1.0002	2.0509	1.4762	1.0005	1.0002
Governments	1.0265	1.3830	1.0000	1.0003	1.0325	1.5067	1.0000	1.0000
Average	1.3452	1.3452	1.0003	1.0003	1.4507	1.4507	1.0003	1.0003

TABLE 6: Internal and External Multipliers of Two-Region Model (Region 4)

	Region 4		External		Rest of Chicago		External	
	Internal Row	Column	Row	Column	Internal Row	Column	Row	Column
Agriculture	1.0568	1.3256	1.0002	1.0011	1.0215	1.2935	1.0001	1.0008
Construction	1.2392	1.5350	1.0008	1.0008	1.1808	1.4696	1.0001	1.0020
Manufacturing (Non-Durable)	1.6191	1.4660	1.0017	1.0009	1.4619	1.3535	1.0012	1.0013
Manufacturing (Durable)	1.6415	1.3859	1.0024	1.0009	1.5062	1.4113	1.0014	1.0015
TCU	1.4552	1.4093	1.0005	1.0008	1.4052	1.3711	1.0014	1.0012
Trades	1.3149	1.3241	1.0008	1.0008	1.2183	1.3153	1.0001	1.0005
FIRE	1.4912	1.4212	1.0002	1.0011	1.6476	1.4604	1.0023	1.0003
Services	1.9687	1.4490	1.0015	1.0010	1.9905	1.4082	1.0035	1.0008
Governments	1.0304	1.5008	1.0000	1.0006	1.0322	1.3816	1.0001	1.0017
Average	1.4241	1.4241	1.0009	1.0009	1.3849	1.3849	1.0011	1.0011

Region 4 has the largest total effect (1.43). The external multipliers, indicating external propagation activities, are considerably smaller than the internal multipliers, the external propagation effects are less than 0.1 percent even in Regions 1 and 4 with the largest average multipliers. Although the external multipliers for some sectors are close to 0.2 percent, (for example, row sum of Manufacturing (Durable) in Region 4, they are still relatively insignificant. This observation implies that the regions within the Chicago metropolitan area are not closely tied to each other in terms of economic activity as defined by movements of goods and services. This is because each metropolitan area is somewhat specialized in specific industries and industries located in geographical subsets of the metropolitan area may be further specialized in a narrower

range of commodities; thus, for smaller geographic areas industries may tend to trade more with sectors located outside the metropolitan area. Thus, the gains from trade expansion in one region are likely to be muted in their impacts on the rest of the metropolitan economy; however, as has been noted many times, the income-consumption linkage tends to dominate at the regional scale especially where analytically important interactions are to be found within input-output or social accounting systems (see, for example, Hewings and Romanos, 1981; Hewings et al., 1989). The next section provides an assessment of the magnitude of these interactions.

### *Commuting and Income Flows*

Although it appears that, interindustry trade between regions within a metropolitan area are less significant with greater geographical disaggregation, it is too premature to conclude that these regions are not economically interdependent. Another form of trade flow, labor, appears to have a significant impact in generating interdependence between regions. The trade flows of labor, that is, commuting, provide significant linkages between regions in two important ways. First, income associated with labor flows moves from one region to another in response to home-work separation; thus, for any region the degree to which income is both earned and accumulated (in the sense of being brought into the household) locally will vary. Second, households receiving this income will chose to spend it on a variety of goods and services, and again, there may be considerable variations in the propensities to consume within the region of residence.

Table 7 shows the flows of labor between the regions. Although the diagonal entries—intraregional commuting—are still the largest, the off-diagonal entries comprise a much larger share than in the case of the trade flows shown in Table 2. For Region 3, there is an almost equal contribution from all four regions to satisfying labor demand in this region. Figure 3 indicates the direction and volume (by thickness of arrow) of the net labor flows between the regions. Region 1 (Loop and North Side) receives positive net labor flows from the remaining three regions. On the other hand, Region 2 (South Side) has negative net labor flows with the rest of the regions. Overall, about 25 percent of labor in the Chicago metropolitan area commutes outside its region of residence. Even more important than the flow of labor is the flow of income back to the region of residence; it turns out that this circulation of income flows underpins the significant interdependence between the regions.

TABLE 7: Commuting (Journey-to-Work) Flows

From/To	Region 1	Region 2	Region 3	Region 4	Total
Region 1	251,416	15,145	39,933	89,623	396,117
Region 2	79,466	137,940	39,411	51,264	308,081
Region 3	43,034	9,626	37,725	28,877	119,262
Region 4	123,093	50,859	36,300	1,378,078	1,588,329
Total	497,008	213,570	153,369	1,547,842	2,411,790

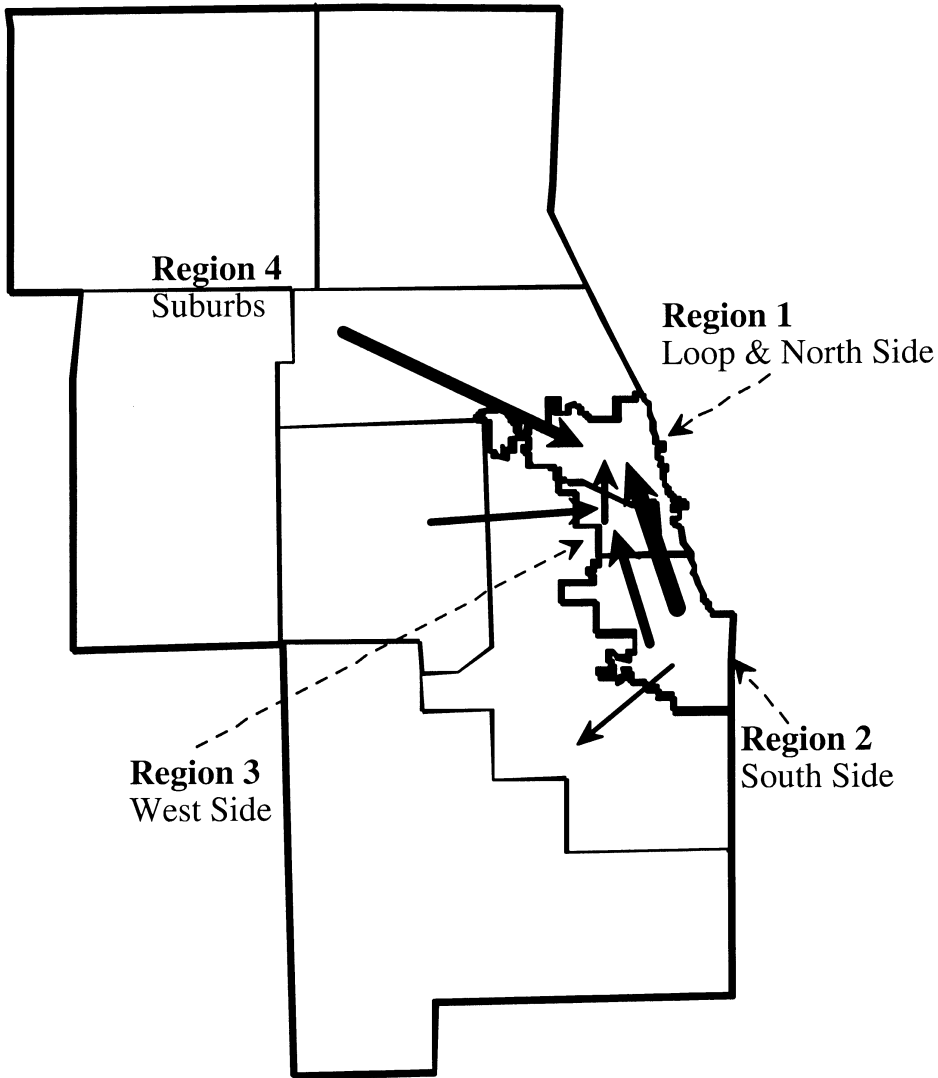


FIGURE 3: Net Labor Flows (Commuting) Between the Regions.

Gross income flows between the regions are shown in Table 8; unlike the aggregated trade flows and the commuting flows, the distribution of these gross income flows presents a different and more complicated pattern. First, the diagonal entries, except for Region 4 (Suburbs), are no longer dominant. A smaller percentage of the income generated remains in the region of origin, this is especially true for Region 3 where only a little over 18 percent of income earned

TABLE 8: Gross Income Flows [\$ 1992 millions]

From/To	Region 1	Region 2	Region 3	Region 4	Total
Region 1	13,413	4,466	1,692	9,032	28,603
Region 2	1,128	5,147	313	3,112	9,700
Region 3	2,106	2,062	1,528	2,603	8,299
Region 4	4,146	2,655	726	82,681	90,208
Total	20,792	14,331	4,258	97,428	136,810

therein remains in the region. Likewise, the dependencies in terms of the origin of income are more varied. For Regions 2 and 3, approximately 65 percent of the income is earned in other regions and brought back to the region; on the other hand, the majority of the income earned in Regions 1 and 4 remains in those regions. Figure 4 indicates the direction and volume (by thickness of arrow) of the net income flows between the regions. These net income flows are primarily a reversal of the net labor flows with two exceptions: (1) income flow from Region 3 to Region 1 and (2) income flow from Region 2 to Region 4. Moreover, the rank of the net flows is different between labor and income flows, indicating differences in the income levels associated with the commuting flows. This observation confirms Mills' (1999) findings of earning inequality in the Chicago metropolitan area.

In the next section, the findings from the aggregate trade, employment, and income flows are integrated to produce Miyazawa's interrelational income multipliers.

#### *Interrelational Income Multipliers*

Table 9 shows Miyazawa's interrelational multipliers, the expression  $K$ , from Equation (2). Here the ripple effects have a similar interpretation to standard income multipliers except that the context is explicitly for interactions across regions rather than between income groups within a region. Further, these income multipliers only explore impacts generated by wage and salary income and thus do not account for other sources of income (see Pyatt, 2000). Their interpretation may be illustrated by reference to the Region 1. For each \$1 of income increase in Region 1, a further \$0.23 of income is generated in Region 1 itself, \$0.11 in Region 2, \$0.03 in Region 3, \$0.44 in Region 4, and \$1.81 in the Chicago metropolitan area as a whole. Among these column sums, Region 2 has the largest income multiplier in the area. It appears that these regional differences in column sums are of little significance; at first, this may seem surprising, especially in view of the differences in the trade relationships in goods and services. However, recall that no specific regional consumption function data were available and thus the relative homogeneity in aggregate income effects is a reasonable outcome. The spatial distribution of the impacts from income change are not homogeneous. In contrast to the column sums that indicate the induced effects originating from each region, the value of row sum



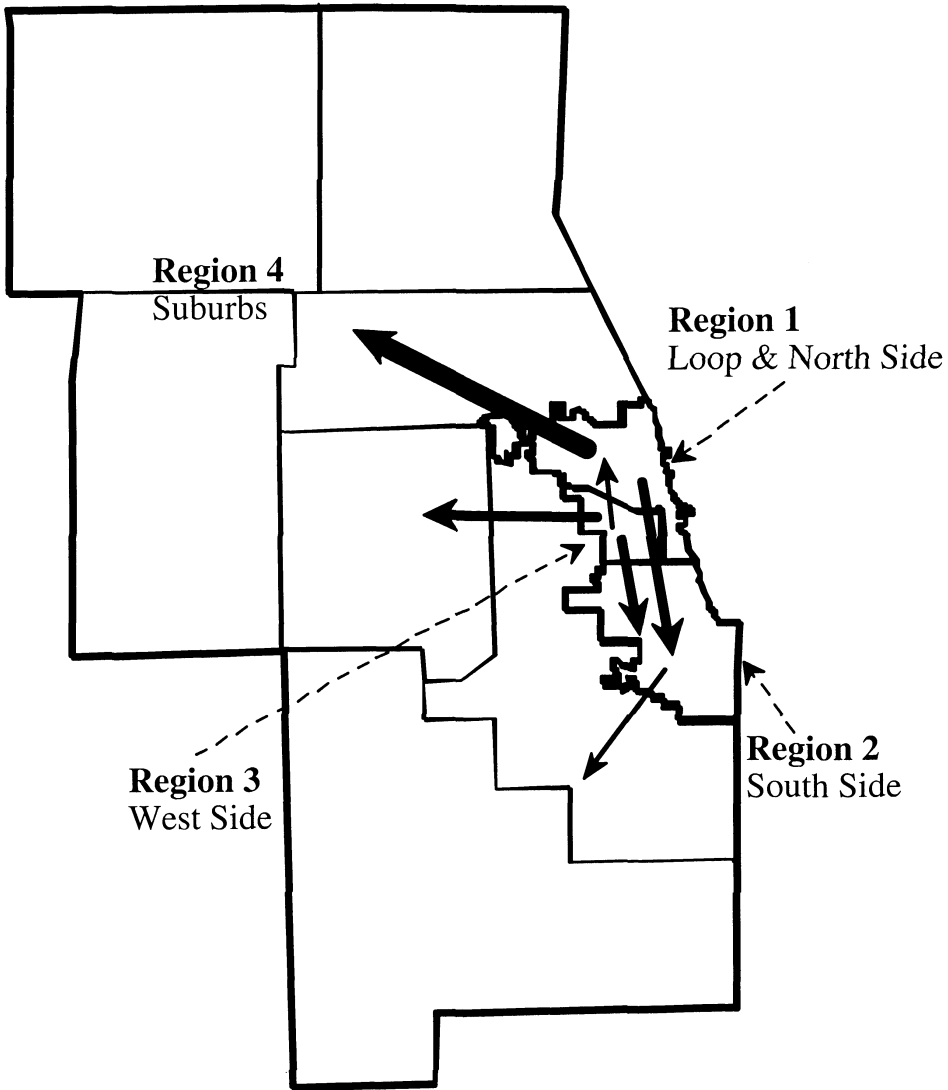


FIGURE 4: Net Income Flows Between the Regions.

shows the induced effects received in each region. These sums reflect the total value of income received in a region as a result of a one dollar change in income in all regions; as may be seen, there are large differences in the row sum values. Region 4 receives induced effects of 3.28, whereas Regions 1 and 2 receive 1.57 and Region 3 receives 1.14. In part, this results from the large differences in region size and also reflects differences in the sources of income.

TABLE 9: Miyazawa's Interrelational Income Multipliers

	Region of Income Origin				Total
	Region 1	Region 2	Region 3	Region 4	
Region of Income Receipt					
Region 1	1.23	0.12	0.16	0.07	1.57
Region 2	0.11	1.28	0.13	0.05	1.57
Region 3	0.03	0.03	1.06	0.01	1.14
Region 4	0.44	0.56	0.50	1.77	3.28
Total	1.81	1.99	1.85	1.90	7.55

To examine the implications of regional demand structure, attention is now directed to income formation by replacing  $\mathbf{f}$  with  $\mathbf{F}$ —a final demand matrix containing each region's final demand separately—in Equation (2). Table 10 shows the coefficients of income inducement per unit of each region's demand. In general, the general features of Table 10 are similar to those of Table 9; however, there is a significant contrast between the induced effects by region of demand origin (column totals) and the induced effects by region of income receipt (average in the last column). Again, the concentration of income formation in Region 4 is observed with much larger differences in coefficient values in this region compared to the remaining regions. This result reflects the fact that the location of demand has a substantial effect in determining regional income generation, especially in the income-receiving base.

Table 11 translates the above results into the percentage dependency of income formation by regional demand. The averages for the Chicago metropolitan area are shown in the last row of the table; 80.5 percent of all the income comes directly and indirectly from the initial expenditure in Region 4, and the contribution of expenditures in Regions 2 and 3 are only 3.8 percent and 2.9 percent, respectively. Moreover, comparing the values of each row, the distribution of income receipt from each region appears to reveal a notable difference between Region 4 and the other regions. In Region 4 a large proportion of the income generated depends on the demand originating in Region 4 itself (86.5 percent) while dependence on Regions 1, 2, and 3 is low. On the other hand, around 50 percent of income in Regions 1, 2, and 3 depends on demand from Region 4. Again, these findings are heavily influenced by the size of Region 4.

## 5. POLICY INTERPRETATIONS

The sparsity of the off-diagonal elements in the trade matrix of goods and services suggest that little benefits are derived from developments in one region on the rest of the economy. However, an examination of the interrelational income multiplier matrix reveals an important finding: often, development programs in inner-city and minority-targeted areas of metropolitan economies are promoted without an understanding or appreciation of the potential spillover effects that they may generate in the rest of the economy. These effects are likely

TABLE 10: Coefficients of Income-inducement Per Unit of Each Region's Demand

	Region of Demand Origin				Average
	Region 1	Region 2	Region 3	Region 4	
Region of Income Receipt					
Region 1	0.301	0.138	0.202	0.087	0.123
Region 2	0.137	0.317	0.188	0.061	0.086
Region 3	0.043	0.032	0.095	0.015	0.022
Region 4	0.582	0.663	0.639	1.055	0.959
Total	1.063	1.150	1.124	1.218	1.190

TABLE 11: Percentage Dependency of Income-Formation by Regional Demand

	Region of Demand Origin				Average
	Region 1	Region 2	Region 3	Region 4	
Region of Income Receipt					
Region 1	35.2%	4.4%	5.1%	55.4%	100.0%
Region 2	23.0%	14.4%	6.8%	55.8%	100.0%
Region 3	27.5%	5.5%	13.2%	53.8%	100.0%
Region 4	8.7%	2.7%	2.1%	86.5%	100.0%
Average	12.8%	3.8%	2.9%	80.5%	100.0%

to be small in terms of goods and services, however, the spillover effects turn out to be very substantial when the income component is examined. Table 9 reveals that a dollar of income generated in Region 2 will create \$0.56 of additional income in Region 4, an amount larger than the combined indirect effect in Regions 1, 2, and 3 (\$0.43). A similar pattern of distribution arises from income expansion in Region 3.

Mills (1999) provides a thoughtful review of initiatives that have been tried and options that need to be considered in the process of attracting business back into the city in general and to minority areas in particular. The findings in our paper suggest considerable benefit accrues to nontarget regions from development initiatives focused on inner city and minority dominated regions. Would this be true if this development was focused around retail activities? Preliminary estimates suggest that between 15 and 20 percent of shopping trips made by residents of the South side of Chicago are outside the region; in large part, the out-of-region trips reflect a dearth of retail opportunities within the region. If new activities were to locate in the region, it is likely that the spillover effects would remain large. Here appeal can be made to the new trade theory with a twist towards retail rather than classic production of goods. At present, most of the retail trade is on goods and services not offered in the region (analogous to interindustry trade); with more retail outlets available locally, a portion of the extra-regional trade will now be substituted by local purchases. However, job

creation in local retail (and, hopefully, other types of economic activities) will raise per capita incomes. Consumers with higher incomes demand greater variety and thus will have a higher probability of searching for retail options outside their region, particularly for higher-order goods. Hence, although there may be a decrease in the volume of expenditures on lower-order goods outside the region, there may be a concomitant increase in the volume of higher-order expenditures elsewhere that may yield a positive balance to the suburban region, for example.

A caveat is in order here; we are not recommending that development in inner cities and minority areas be based on the gains that may accrue outside the region. What we are suggesting is that spillovers do occur, they appear to be sizeable, and this information should be used to promote development strategies to impress on residents and policymakers outside the targeted areas that it is possible to realize gains in all regions. The type of reasoning propounded in support of free trade should resonate in the thinking of intrametropolitan-scale development.

## 6. CONCLUSIONS

In this paper, economic interdependence between the four regions in the Chicago metropolitan area is analyzed. Employing Miyazawa's extended input-output framework, the model constructed illustrates the spatial economic structure of the Chicago area. Based on the above findings, as Stahl (1987) emphasized, intraurban location of employment generates clear distinctions among industrial sectors. The differences in the location of employment among sectors not only creates multiple employment centers but generates a complex commuting pattern, and thus income flows, between employment and residential locations within a metropolitan area.<sup>5</sup> The Miyazawa multipliers that take into account the interactions of income flows and consumption behavior, reveal further detailed relationships in income formation between the regions. In this study, the model clearly illustrates the systematic interdependence of income formation among the regions, providing a mechanism to trace the path of income formation origin to demand region.

The analysis also reveals that interdependence among regions varies depending on whether the focus is on production, employment, or income. While the interindustry relationship generates circulation of economic activity and hence creates impacts outside the region of original stimulus, the size of these impacts is relatively small. The greatest source of this variation originates in the journey-to-work trips, that is, commuting. In this context, Region 3 is more dependent on the other regions in that a very higher percentage of the workforce commutes into the region. On the other hand, Region 4 (the suburbs) is more self-contained; however, this may be a reflection of the larger size of the region.

---

<sup>5</sup>There is virtually no difference in highway commuting times inbound to or outbound from the CBD of Chicago during the morning and evening rush hours.

Cole's (1999) study also found a similar asymmetry between the inner-city community and the rest of the metropolitan region in Buffalo, New York.

Given the above findings, for further analysis it is important to take into account differences in consumption behavior across income groups. Miyazawa's extended input-output framework can be further employed for this task, using value-added and consumption coefficients for  $r$ -fold income group in matrices  $\mathbf{V}$  and  $\mathbf{C}$ . Furthermore, there is a strong implication from the findings of this paper for interregional and international trade theory. As discussed in Sections 4 and 5, interindustry interactions, namely trade, among four regions in the Chicago metropolitan area are fairly weak with only small external multipliers; however, overall economic interdependence is very strong, originating mainly from journey-to-work trips. In providing a connection with the hollowing-out phenomenon in the Chicago metropolitan area by Hewings et al. (1998), the findings in this paper raise an interesting question: Does geographic size matter in trade? In order to answer this question, a comparative analysis between intrametropolitan, interregional trade, and international trade will be an important next step to begin this exploration.

Economic development in inner-city communities in U.S. metropolitan areas has always been hampered by the absence of strategic information about the interdependence between inner-city and other regions within a metropolitan area. The analytical framework and the results in this paper offer an important input to the planning and development process of inner-city communities.

## REFERENCES

- Bon, Ranko. 1975. *Some Conditions of Macroeconomic Stability in Multiregional Models*. Department of Transportation, Report No. 10. Washington DC: University Research Program, U.S. Department of Transportation.
- . 1984. "Comparative Stability Analysis of Multiregional Input-Output Models: Column, Row, and Leontief-Strout Gravity Coefficient Models," *Quarterly Journal of Economics*, 99, 791–825.
- Chenery, Hollis B. 1953. "Regional Analysis," in H.B. Chenery, P.G. Clark, and V.C. Pinna (eds.), *The Structure and Growth of the Italian Economy*. Rome: U.S. Mutual Security Agency, pp. 97–129.
- Chicago Area Transportation Study. 1994. *CATS 1990 Household Travel Survey*, Technical Documentation for the Household, Person and Trip Files.
- Cole, Sam. 1994. "A Community Accounting Matrix for Buffalo's East Side Neighborhood," *Economic Development Quarterly*, 8, 107–126.
- . 1999. "In the Spirit of Miyazawa: Multipliers and the Metropolis," in G.J.D. Hewings, M. Sonis, M. Madden, and Y. Kimura (eds.), *Understanding and Interpreting Economic Structure*. New York: Springer-Verlag, pp. 263–286.
- Goldsmith, William and Edward Blakeley (1992) *Separate Societies: Poverty and Inequality in US Cities*. Philadelphia: Temple University Press.
- Harrison, Bennett. 1974. "Ghetto Economic Development," *Journal of Economic Literature*, 12, 1–37.
- Hewings, Geoffrey J.D., Manuel A.R. da Fonseca, Joaquim J.M. Guilhoto, and Michael Sonis. 1989. "Key Sectors and Structural Change in the Brazilian Economy: A Comparison of Alternative Approaches and their Policy Implications," *Journal of Policy Modeling*, 11, 67–90.
- Hewings, Geoffrey J.D. and Michael C. Romanos 1981. "Simulating Less Developed Regional Economies Under Conditions of Limited Information," *Geographical Analysis* 13, 373–390.
- Hewings, Geoffrey J.D., Michael Sonis, Jiemin Guo, Philip R. Israilevich, and Graham R. Schindler. 1998. "The Hollowing-Out Process in the Chicago Economy, 1975–2011," *Geographical Analysis*, 30, 217–233.

- Hewings, Geoffrey J.D., Michael Sonis, Moss Madden, and Yoshio Kimura, eds. 1999. *Understanding and Interpreting Economic Structure*. New York: Springer-Verlag.
- Illinois Department of Employment Security. 1996. *Where Workers Work in the Chicago Metro Area*, Supplemental Report: 1993–1995.
- Isard, Walter and David F. Bramhall. 1960. "Gravity, Potential and Spatial Interaction Models," in W. Isard, (ed.), *Methods of Regional Analysis*. Cambridge, MA: MIT Press, pp 493–568.
- Israilevich, Philip R., Geoffrey J.D. Hewings, Michael Sonis, and Graham R. Schindler. (1997) "Forecasting Structural Change with a Regional Econometric Input-Output Model," *Journal of Regional Science*, 37, 565–590.
- Leontief, Wassily and Alan Strout. 1963. "Multiregional Input-Output Analysis," in T. Barna (ed.), *Structural Interdependence and Economic Development*, London: MacMillan, pp. 119–150.
- Mellor, E. 1972. "Costs and Benefits of Public Goods for a Ghetto: A Case Study," in K. Boulding (ed.), *Transfers in an Urbanized Society*. Belmont, MA: Wadsworth.
- Mills, Edwin S. 1999. "Earnings Inequality and Central-City Development," *Federal Reserve Bank of New York Economic Policy Review*, 5, 133–142.
- Miyazawa, Ken'ichi. 1976. *Input-output Analysis and the Structure of Income Distribution*. New York, NY: Springer-Verlag.
- Moses, Leon N. 1955. "The Stability of Interregional Trading Patterns and Input-Output Analysis," *American Economic Review*, 45, 803–832.
- Polenske, Karen R. 1970. "An Empirical Test of Interregional Input-Output Models Estimation of 1963 Japanese Production," *American Economic Review*, 60, 76–82.
- Pyatt, Graham. 2001. "Some Early Multiplier Models of the Relationship between Income Distribution and Production Structure," *Economic Systems Research*.
- Schaffer, Ronald. 1973. *Income Flows in Urban Poverty Areas*, Lexington, MA: Heath.
- Sonis, Michael and Geoffrey J.D. Hewings. 1993. "Hierarchies of Regional Sub-Structures and Their Multipliers within Input-Output Systems: Miyazawa Revisited," *Hitotsubashi Journal of Economics*, 34, 33–44.
- Stahl, Konrad. 1987. "Theories of Urban Business Location," in E. Mills (ed.), *Handbook of Regional and Urban Economics: Volume 2*, New York, NY: North-Holland, pp. 759–820.
- Toyomane, N. 1988. *Multiregional Input-Output Models in Long-Run Simulation*, Dordrecht: Kluwer Academic.
- Victorisz, Tom and Bennett Harrison. 1970. *The Economic Development of Harlem*. New York: Praeger.

## APPENDIX

For simplicity, consider a two-region ( $p$  and  $q$ ) and two-sector (1 and 2) system. The location quotient of sector  $i$  in region  $p$  can be written as

$$LQ_i^p = \frac{e_i^p}{e_1^p + e_2^p} \bigg/ \frac{(e_i^p + e_i^q)}{(e_1^p + e_2^p + e_1^q + e_2^q)}$$

Without loss of generality, suppose that the sum of the total employment is unity. Thus

$$e_1^p + e_2^p + e_1^q + e_2^q = 1$$

Hence, the location quotient now becomes the following simpler form

$$LQ_i^p = \frac{e_i^p}{e_1^p + e_2^p} \bigg/ (e_i^p + e_i^q) = \frac{e_i^p}{(e_1^p + e_2^p)(e_i^p + e_i^q)}$$

Then, the export share of sector  $i$  in region  $p$  is

$$ex_i^p = 1 - \frac{1}{LQ_i^p} = 1 - \frac{(e_1^p + e_2^p)(e_i^p + e_i^q)}{e_i^p}$$

However, this export share is only calculated if the sector  $i$  in region  $p$  has a location quotient larger than unity,  $LQ_i^p > 1$ ; therefore, the relationship  $ex_i^p \geq 0$  always holds.

There are only two regions in this system so the row trade coefficient becomes the export share of sector  $i$  in region  $p$

$$r_i^{pq} = ex_i^p = 1 - \frac{(e_1^p + e_2^p)(e_i^p + e_i^q)}{e_i^p}$$

because  $ex_i^p \geq 0$ ,  $r_i^{pq} \geq 0$ . Given the definition of the row trade coefficient

$\left( \sum_q r_i^{pq} = 1 \right)$ , the row intraregional trade coefficient becomes

$$r_i^{pp} = 1 - r_i^{pq} = \frac{(e_1^p + e_2^p)(e_i^p + e_i^q)}{e_i^p}$$

and

$$r_i^{qq} = 1 - r_i^{qp} = \frac{(e_1^q + e_2^q)(e_i^p + e_i^q)}{e_i^q}$$

Then these row trade coefficients are transformed into the column trade coefficients as follows

$$c_i^{pq} = \frac{r_i^{pq}(e_1^p + e_2^p)}{r_i^{pq}(e_1^p + e_2^p) + r_i^{qq}(e_1^q + e_2^q)}$$

because  $r_i^{pq} \geq 0$  and  $e_i^p \geq 0$  for  $\forall p, q, i$ ,  $c_i^{pq} \geq 0$  for  $\forall p, q, i$ .