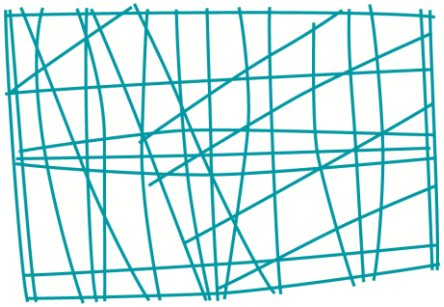


NEREUS

Núcleo de Economia Regional e Urbana
da Universidade de São Paulo

The University of São Paulo
Regional and Urban Economics Lab

Lecture 3: Factor Analysis

Prof. Eduardo A. Haddad Prof. Vinicius Vale

Specific use

How to identify the development potential of municipalities in the Colombian Pacific Region?

Database for 178 municipalities includes 66 variables classified in four specific dimensions:

- Economic, environmental, social, and institutional

Is it possible to **summarize** these variables in one single indicator for all the municipalities?

General use

Econometrics:

Create variables that, by construction, are uncorrelated (multicollinearity)

Regional and urban analysis:

Build an indicator that allows classifying regions and/or cities

Two types of factor analysis

Exploratory:

- ✓ It is exploratory when you do not have a pre-defined idea of the structure or how many dimensions are in a set of variables

Confirmatory:

- ✓ It is confirmatory when you want to test specific hypothesis about the structure or the number of dimensions underlying a set of variables (i.e. in your data you may think there are two dimensions and you want to verify that)

"Data reduction" – synthesis

Build indexes from a set of *k* variables, X_j

Transformation through a **linear combination** of the *k* variables, X_j

$$Z_1 = a_{11}X_1 + a_{12}X_2 + \ldots + a_{1k}X_k$$

$$Z_2 = a_{21}X_1 + a_{22}X_2 + \ldots + a_{2k}X_k$$

$$\ldots$$

$$Z_k = a_{k1}X_1 + a_{k2}X_2 + \ldots + a_{kk}X_k$$

What weights(a_{ij}) to use?

Variables Z_i should be orthogonal

Variables Z_i should be calculated so that Z_1 (the first factor) explains the largest share of total variance of variables X_j; Z_2 the second largest share, etc.

Objectives

To find a way of condensing (summarize) the information contained in a list of original variables into a smaller set of variables (factors), **without loss of information**

To group variables based on their correlations, i.e. all the variables within a particular group should be highly correlated among them, but should present, in relative terms, low correlation with variables from the other groups

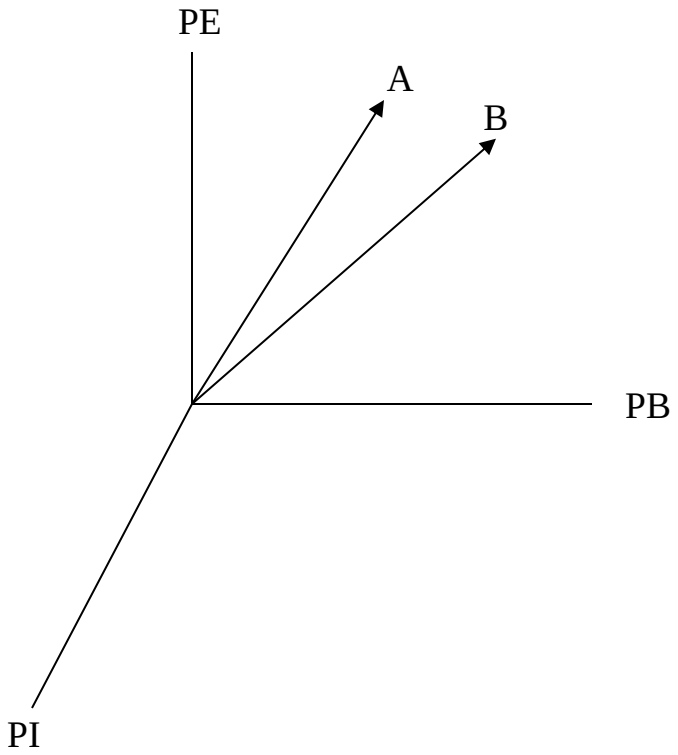
Economic and social indicators for the states of Brazil's Northeast (1980)

Estados	Variáveis							
	X1	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**X8**
MA	29.97	3978	42.00	16488	5848	528	25.13	69.4
PI	20.67	2130	42.55	11006	4225	313	31.93	59.7
CE	33.09	5271	48.72	83926	16868	1310	40.81	43.2
RN	37.20	1892	50.89	34408	5520	595	47.57	38.2
PB	27.50	2763	46.43	36271	9374	716	42.06	48.1
PE	47.59	6125	51.65	121129	19920	3223	54.47	40.8
AL	38.85	1975	40.23	36663	6058	731	39.78	53.2
SE	41.14	1135	45.59	18556	3144	410	46.12	42.9
BA	53.72	9417	49.71	104576	17508	7823	41.18	49.2

X1 – Per capita income in 1980, in Cz$
X2 – Population in 1980 1980, in 1,000
X3 – Literacy rate, in 1980
X4 – Employment in manufacturing, on 31.12.1980
X5 – Number of hospital beds, in 1980
X6 – Total consumption of electricity, in 1980
X7 – Urbanization rate, in 1980
X8 – Share of labor force in primary sector, in 1980

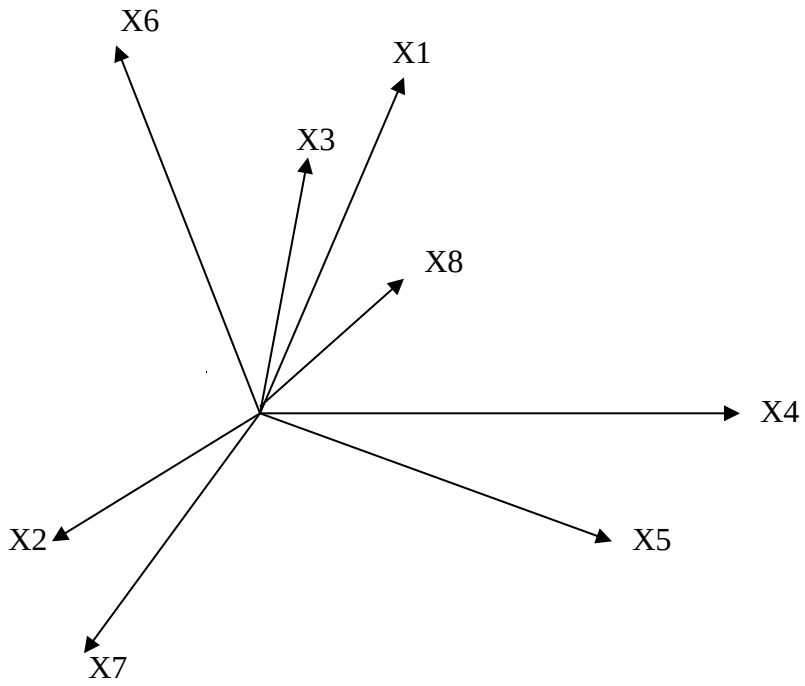
Intuition

Graphical representation of (standardized) values of Per capita income (A) and urbanization rate (B) for three states



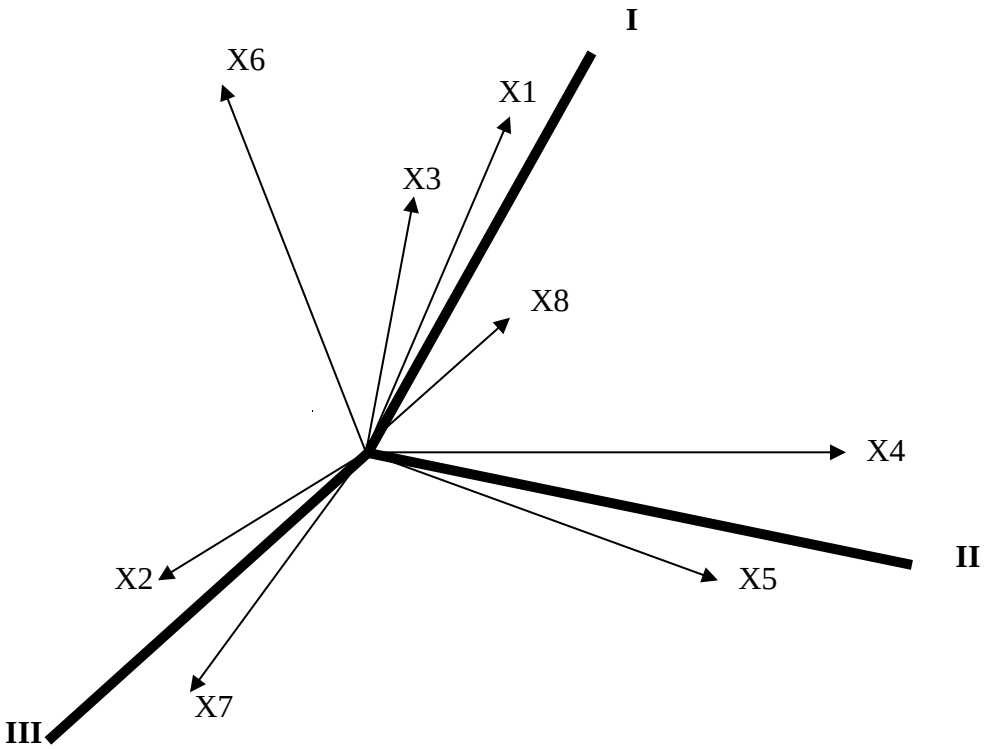
For all variables...

The angles between vectors measure the relation among characteristics that are represented by the vectors; thus, the; therefore, the **smaller** the angle, the **greater** the association between the two characteristics or variables



Possible factors

In this example, we can distinguish three *groupings of variables* (factors): (i) variables X6, X3, X1 e X8; (ii) variables X4 e X5; and (iii) variables X2 e X7



Factor loading

Factor analysis is about defining these axes or factors

The three factors would represent the variability of characteristics that are being used to represent the regions

The projections of each one of the eight variables on each axis define the relation of each variable with each factor

These projections are named **factor loadings**

Stages

1. Build the correlation matrix
2. Extract the initial factors
3. Rotate the factors
4. Calculate the factor scores
5. Group the factor scores

1. Build the correlation matrix

Matrix R

	X1	X2	X3	X4	X5	X6	X7	X8
X1	1.000							
X2	0.615	1.000						
X3	0.538	0.505	1.000					
X4	0.713	0.837	0.714	1.000				
X5	0.560	0.860	0.668	0.972	1.000			
X6	0.765	0.913	0.492	0.755	0.706	1.000		
X7	0.595	0.105	0.735	0.565	0.432	0.233	1.000	
X8	-0.476	-0.050	-0.773	-0.475	-0.369	-0.159	-0.930	1.000

2. Extract the initial factors

Communality

	Fator 1	Fator 2	Fator 3	Fator 4	Fator 5	Fator 6	Fator 7	h2
X1	**0.841**	0.068	**0.515**	0.070	0.056	-0.105	-0.004	**0.996**
X2	**0.781**	**0.616**	-0.060	-0.077	-0.004	-0.029	0.017	1.000
X3	**0.837**	-0.311	-0.263	-0.348	0.095	-0.027	-0.003	0.997
X4	**0.949**	0.167	-0.149	0.205	0.033	-0.006	-0.033	0.995
X5	**0.881**	0.280	-0.326	0.193	-0.039	-0.005	0.018	1.000
X6	**0.797**	0.482	0.286	-0.175	-0.077	0.112	-0.008	0.998
X7	**0.698**	**-0.684**	0.008	0.130	0.072	0.122	0.013	0.992
X8	**0.641**	**0.742**	0.028	0.039	0.172	0.047	0.002	0.996
Eingevalue	5.228	1.838	0.549	0.260	0.055	0.042	0.002	
% da variância	**0.653**	**0.230**	**0.069**	**0.033**	**0.007**	**0.005**	**0.000**	
% var acum	0.653	0.883	0.952	0.984	0.991	0.997	0.997	

Factor loadings: we usually use factor loadings to label common factors

Definitions

The sum of the squared factor loadings for all factors for a given variable (row) is the variance in that variable accounted for by all the factors, and this is called the **communality**

The **eigenvalue** is a measure of how much of the variance of the observed variables a factor explains

% of variance gives the ratio, expressed as a percentage, of the variance accounted for by each factor to the total variance in all of the variables

3. Rotate the factors

Objective: better define the relations between variables and factors

An **orthogonal rotation** method minimizes the number of variables that have high loadings on each factor. This method simplifies the interpretation of the factors.

A **varimax rotation** is used to simplify the expression of a particular sub-space in terms of just a few major items each (the actual coordinate system is unchanged, it is the orthogonal basis that is rotated to align with those coordinates).

Results from a *varimax rotation*

Cargas Fatoriais - Rotacionadas

	Fator 1	Fator 2	Fator 3	Fator 4
X1	0.329	0.488	**0.789**	-0.032
X2	**0.854**	-0.092	0.475	0.218
X3	0.451	**0.690**	0.136	**0.545**
X4	**0.859**	0.390	0.306	0.029
X5	**0.948**	0.271	0.152	0.051
X6	0.619	0.021	**0.735**	0.170
X7	0.140	**0.952**	0.190	-0.018
X8	-0.086	**-0.975**	-0.090	-0.086
Eingevalue	3.088	2.806	1.568	0.386
% da variância	**0.386**	**0.351**	**0.196**	**0.048**
% var acum	0.386	0.737	0.933	0.981

4. Calculate the factor scores

After rotating the factors, it is possible to generate factor scores for each unit of observation

Calculation procedure:

- Standardize the original data
- Calculate the factor loadings
- Calculate the factor scores

Original data

Indicadores sociais e econômicos para os estados Nordestinos em 1980

Estados	Variáveis							
	X1	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**X8**
MA	29.97	3978	42.00	16488	5848	528	25.13	69.4
PI	20.67	2130	42.55	11006	4225	313	31.93	59.7
CE	33.09	5271	48.72	83926	16868	1310	40.81	43.2
RN	37.20	1892	50.89	34408	5520	595	47.57	38.2
PB	27.50	2763	46.43	36271	9374	716	42.06	48.1
PE	47.59	6125	51.65	121129	19920	3223	54.47	40.8
AL	38.85	1975	40.23	36663	6058	731	39.78	53.2
SE	41.14	1135	45.59	18556	3144	410	46.12	42.9
BA	53.72	9417	49.71	104576	17508	7823	41.18	49.2
Média	37	3854	46	51447	9829	1739	41	49
Desvio Padrão	10	2671	4	40945	6474	2450	9	10

Standardized data

Matriz de Dados Normalizados - Z

Estados	Variáveis							
	Z1	**Z2**	**Z3**	**Z4**	**Z5**	**Z6**	**Z7**	**Z8**
MA	-0.65	0.05	-1.07	-0.85	-0.62	-0.49	-1.85	2.00
PI	-1.56	-0.65	-0.94	-0.99	-0.87	-0.58	-1.06	1.03
CE	-0.35	0.53	0.56	0.79	1.09	-0.17	-0.02	-0.62
RN	0.06	-0.73	1.08	-0.42	-0.67	-0.47	0.77	-1.12
PB	-0.90	-0.41	0.00	-0.37	-0.07	-0.42	0.12	-0.13
PE	1.07	0.85	1.26	1.70	1.56	0.61	1.57	-0.86
AL	0.22	-0.70	-1.50	-0.36	-0.58	-0.41	-0.14	0.38
SE	0.44	-1.02	-0.20	-0.80	-1.03	-0.54	0.60	-0.65
BA	1.67	2.08	0.80	1.30	1.19	2.48	0.02	-0.02

Factor scores

Matriz de Factor Score para os estados Nordestinos

	Fator 1	Fator 2	Fator 3	Fator 4
MA	-2.71	-5.28	-1.89	-0.83
PI	-3.75	-3.99	-2.82	-0.84
CE	2.24	1.35	0.38	0.53
RN	-1.20	2.31	-0.48	0.38
PB	-1.26	-0.33	-1.30	-0.14
PE	5.26	4.75	3.00	1.12
AL	-2.37	-1.67	-0.93	-1.12
SE	-2.68	0.77	-0.79	-0.47
BA	6.47	2.09	4.83	1.36
Média + 1DP	3.74	3.19	2.43	0.89
Média	0.00	0.00	0.00	0.00
Média - 1DP	-3.74	-3.19	-2.43	-0.89

5. Group the factor scores

Fator 1		***Clusters***
Alto	> 3.74	PE, BA
Média	<3.74 e >-3.74	MA, CE, RN, PB, AL, SE
Baixo	<-3.74	PI

Fator 2		***Clusters***
Alto	> 3.19	PE
Média	<3.19 e >-3.19	CE, RN, AL, SE, BA
Baixo	<-3.19	PI, MA

Fator 3		***Clusters***
Alto	> 2.43	PE, BA
Média	<2.43 e >-2.43	MA, CE, RN, PB,AL, SE
Baixo	<-2.43	PI

"Índice de Potencial de Desarrollo Municipal: Región Pacífico de Colombia"

Objetivo:

Calcular el potencial de desarrollo de los municipios colombianos y, a partir de ellos, construir tipologías que permitan delimitar áreas económicas deprimidas y desarrolladas.

¿El potencial diferenciado puede explicar los diferenciales de ingreso entre los municipios colombianos?

Metodología

El método en tres pasos:

- ✓ Aplicar análisis factorial (Stata)
- ✓ Construir un factor ponderado endógenamente (Excel)
- ✓ Análisis exploratorio de datos espaciales (GeoDa)

Factor ponderado

El factor ponderado, es decir, el factor de síntesis, se elaborará de la siguiente manera:

$$FP_i = (varF1/varTotal)* F1_i + (varF2/varTotal) *F2_i + (varF3/varTotal) *F3_i + (varF4/varTotal) *F4_i$$

1. Build the correlation matrix

Indicador de Potencial de Desarrollo Municipal: Región Pacífico de Colombia
Colombia

Correlación

		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
1	X1_VA	1															
2	X2_POP	0.982	1														
3	Y3_IND_MAN	0.708	0.573	1													
4	Y4_SERV	0.992	0.996	0.615	1												
5	INGRESOS	0.992	0.984	0.647	0.995	1											
6	AGRIC_AREA	0.076	0.051	0.120	0.058	0.052	1										
7	USO1_ADEC	0.077	0.044	0.127	0.061	0.061	0.678	1									
8	USO2_SOBRE	0.010	0.022	0.004	0.015	0.021	0.585	0.826	1								
9	PRECIPITAC	0.074	0.043	0.129	0.057	0.060	0.589	0.545	0.431	1							
10	COBERT_PRE	0.204	0.178	0.250	0.180	0.160	0.106	0.006	0.021	0.072	1						
11	COBERT_SEC	0.100	0.066	0.164	0.078	0.067	0.480	0.358	0.244	0.316	0.559	1					
12	IDH2_EDUC	0.122	0.096	0.148	0.106	0.094	0.537	0.443	0.312	0.379	0.437	0.900	1				
13	IDH3_INGR	0.234	0.160	0.408	0.178	0.170	0.577	0.476	0.399	0.424	0.415	0.584	0.553	1			
14	DESEMP_FIS_3	0.310	0.249	0.440	0.269	0.278	0.246	0.190	0.096	0.305	0.162	0.240	0.210	0.391	1		
15	DESEMP_INT_4	0.125	0.108	0.177	0.108	0.104	0.469	0.346	0.238	0.435	0.030	0.219	0.224	0.233	0.355	1	
16	DESEMP_INT_5	0.255	0.220	0.325	0.227	0.223	0.275	0.200	0.108	0.313	0.185	0.262	0.254	0.317	0.600	0.581	1

2. Extract the initial factors

Total variance accounted by each factor. The sum of all eigenvalues = total number of variables. When negative, the sum of eigenvalues = total number of factors (variables) with positive eigenvalues.

Kaiser criterion suggests to retain those factors with eigenvalues equal or higher than 1.

Indicador de Potencial de Desarrollo Municipal: Región Pacífico de Colombia Colombia

Factor analysis/correlation Number of obs = 178
Method: component factors Retained factors = 4
Rotation: (unrotated) Number of params = 58

Factor	*Eigenvalue*	*Difference*	*Proportion*	*Cumulative*
Factor1	**5.4107**	**1.7364**	**0.4491**	**0.4491**
Factor2	**3.6744**	**2.2942**	**0.3050**	**0.7541**
Factor3	**1.3802**	**0.3171**	**0.1146**	**0.8687**
Factor4	**1.0630**	**0.4898**	**0.0882**	**0.9569**
Factor5	0.5732	0.3535	0.0476	1.0045
Factor6	0.2197	0.0667	0.0182	1.0227
Factor7	0.1530	0.0820	0.0127	1.0354
Factor8	0.0710	0.0633	0.0059	1.0413
Factor9	0.0078	0.0079	0.0006	1.0419
Factor10	-0.0001	0.0005	0.0000	1.0419
Factor11	-0.0006	0.0207	-0.0001	1.0419
Factor12	-0.0213	0.0484	-0.0018	1.0401
Factor13	-0.0697	0.0256	-0.0058	1.0343
Factor14	-0.0953	0.0243	-0.0079	1.0264
Factor15	-0.1196	0.0790	-0.0099	1.0165
Factor16	-0.1986	.	-0.0165	1.0000

Since the sum of eigenvalues = total number of variables. Proportion indicate the relative weight of each factor in the total variance. For example, 5.4107/12.0475=0.4491. The first factor explains 44.91% of the total variance

Cumulative shows the amount of variance explained by n+(n-1) factors. For example, factor 1 and factor 2 account for 75.41% of the total variance.

3. Rotate the factors

By default the rotation is varimax which produces orthogonal factors. This means that factors are not correlated to each other.
This setting is recommended when you want to identify variables to create indexes or new variables without inter-correlated components

Same description as in the previous slide with new composition between the two factors. Still the four factors explain 95.96% of the total variance observed.

Factor analysis/correlation Number of obs = 178
Method:component factors Retained factors = 4
Rotation: orthogonal varimax (Kaiser off) Number of params = 58

Factor	*Variance*	*Difference*	*Proportion*	*Cumulative*
Factor1	4.5146	1.8274	0.3747	0.3747
Factor2	2.6872	0.0631	0.2230	0.5978
Factor3	2.6241	0.9218	0.2178	0.8156
Factor4	1.7023	.	0.1413	0.9569

3. Rotate the factors

The pattern matrix here offers a clearer picture of the relevance of each variable in the factor. Factor1 is mostly defined by "economic", factor2 by "environmental", Factor 3 by "social", and Factor 4 by "institutional"

Uniqueness is the variance that is 'unique' to the variable and not shared with other variables. It is equal to 1 – communality (variance that is shared with other variables). For example, 3.42% of the variance in "X2_POP" is not shared with other variables in the overall factor model. On the contrary "PRECIPITAC" has high variance not accounted by other variables (54.23%). Notice that the greater "uniqueness" the lower the relevance of the variable in the factor model.

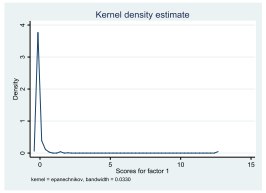
Rotated factor loadings (pattern matrix) and unique variances

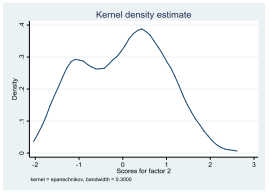
Variable	*Variable*	*Factor1*	*Factor2*	*Factor3*	*Factor4*	*Uniqueness*
X1_VA	Valor Agregado	**0.9929**	0.0159	0.0668	0.0966	0.0002
X2_POP	Población	**0.9821**	0.0148	0.0267	0.0198	0.0342
Y3_IND_MAN	Valor Agregado - Industria Manufacturera	**0.6705**	-0.0436	0.1805	0.4104	0.3475
Y4_SERV	Valor Agregado - Actividades de servicios	**0.9912**	0.0211	0.0369	0.0364	0.0144
INGRESOS	Total de ingresos tributarios que obtiene el estado a traves de impuestos	**0.9916**	0.0187	0.0203	0.0506	0.0134
AGRIC_AREA	Valor agregado agricultura / Área dedicada a cultivos agricolas	0.0154	**0.7024**	0.3530	0.2373	0.3255
USO1_ADEC	Uso del Suelo Rural - Adecuado	-0.0436	**-0.8727**	-0.2014	-0.0857	0.1886
USO2_SOBRE	Uso del Suelo Rural - Sobreutilización	-0.0249	**0.8326**	0.1148	-0.0222	0.2926
PRECIPITAC	Precipitación	-0.0182	**-0.5503**	-0.2112	-0.3315	0.5423
COBERT_PRE	Cobertura del sistema educativo en el nivel Prescolar	0.1588	-0.1486	**0.6237**	0.0714	0.5586
COBERT_SEC	Cobertura del sistema educativo en el nivel Básica Secundaria y Media	0.0272	0.1852	**0.9120**	0.0923	0.1246
IDH2_EDUC	Índice de Educacion	0.0526	0.3031	**0.8424**	0.0554	0.1926
IDH3_INGR	Índice de Ingreso	0.1652	0.3779	**0.6074**	0.2735	0.3862
DESEMP_FIS_3	Desempeño Fiscal - Ingresos que corresponden a transferencias	-0.2399	-0.0920	-0.1748	**-0.6271**	0.5102
DESEMP_INT_4	Desempeño Integral - Gestión Administrativa y Fiscal	0.0531	0.3470	0.0641	**0.5911**	0.5232
DESEMP_INT_5	Desempeño Integral - Fiscal	0.1721	0.1152	0.1659	**0.7155**	0.4176

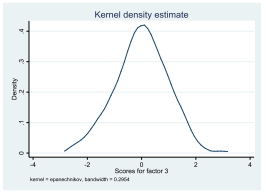
Factor loadings are the weights and correlations between each variable and the factor. The higher the load the more relevant in defining the factor's dimensionality. A negative value indicates an inverse impact on the factor. Here, four factors are retained because both have eigenvalues over 1.

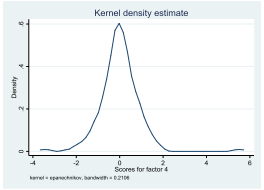
4. Calculate the factor scores

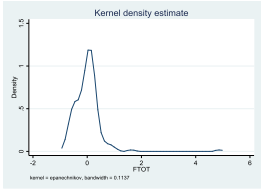
Variable	Obs	Mean	Std. Dev.	Min	Max
Factor1	178	0.000	0.999	-0.379	12.627
Factor2	178	0.000	0.940	-1.739	2.317
Factor3	178	0.000	0.957	-2.550	2.876
Factor4	178	0.000	0.879	-3.451	5.505
FTOT	178	0.000	0.525	-0.821	4.868











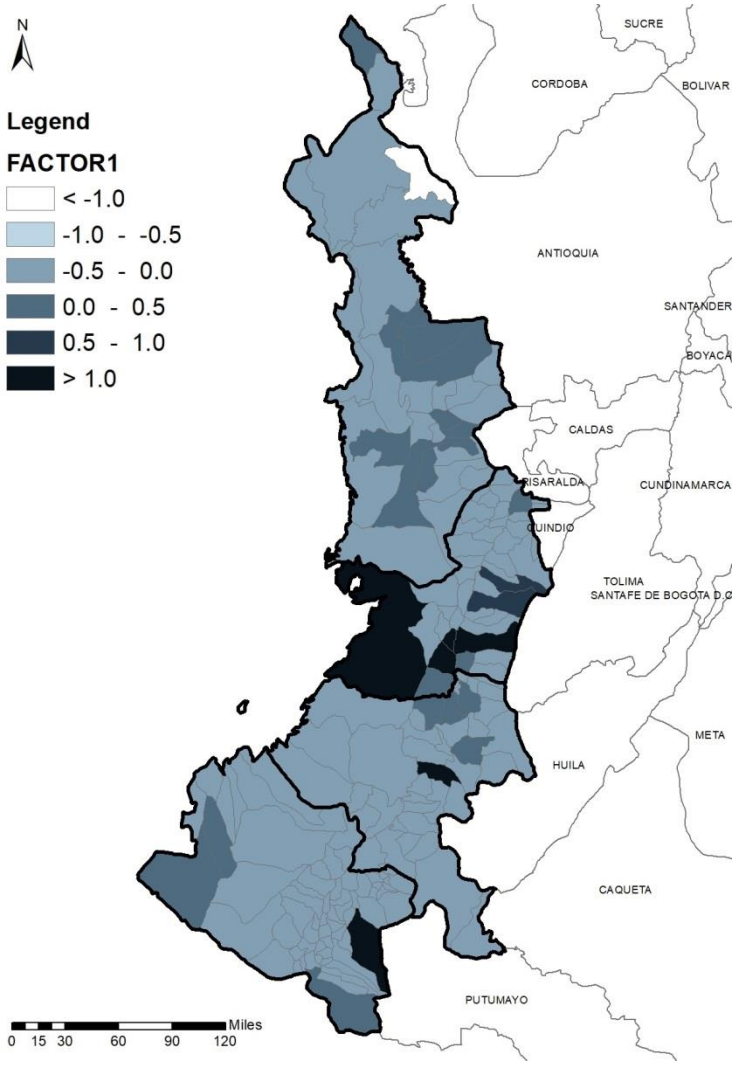
5. Group the factor scores

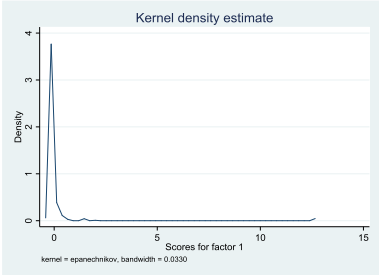
Factor 2 – "AMBIENTAL"		Factor 1 – "ECONÓMICA" (+) / Factor 3 – "SOCIAL" (+)	Factor 1 – "ECONÓMICA" (+) / Factor 3 – "SOCIAL" (-)	Factor 1 – "ECONÓMICA" (-) / Factor 3 – "SOCIAL" (+)	Factor 1 – "ECONÓMICA" (-) / Factor 3 – "SOCIAL" (-)
	(+)	REGIÓN 1 REGIÓN 2	REGIÓN 3 REGIÓN 4	REGIÓN 9 REGIÓN 10	REGIÓN 11 REGIÓN 12
	(-)	REGIÓN 5 REGIÓN 6	REGIÓN 7 REGIÓN 8	REGIÓN 13 REGIÓN 14	REGIÓN 15 REGIÓN 16

OBS. Factor 4 – "INSTITUCIONAL" - encima de la media / abajo de la media

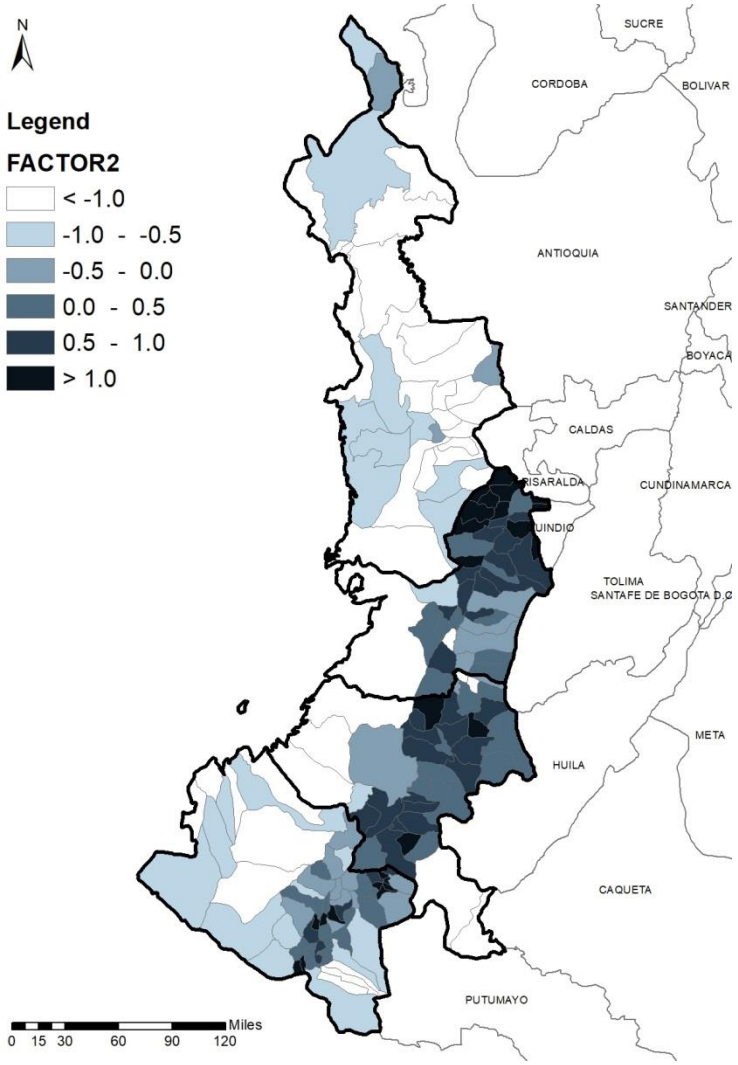
Fuente: Adaptado de Haddad et al. (2018).

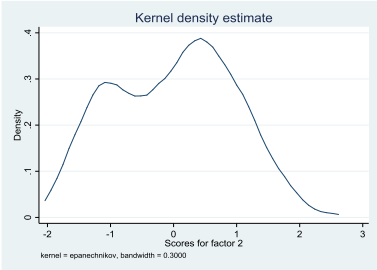
Factor 1 – Dimensión económica



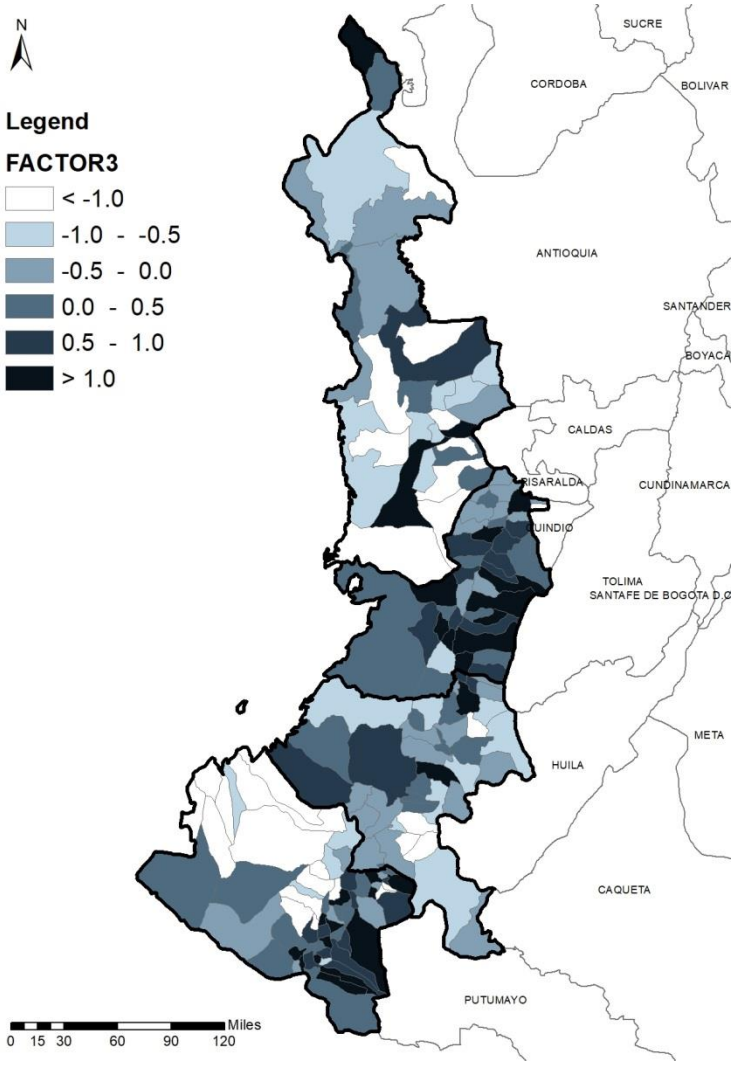


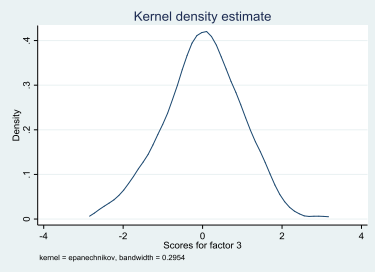
Factor 2 – Dimensión ambiental



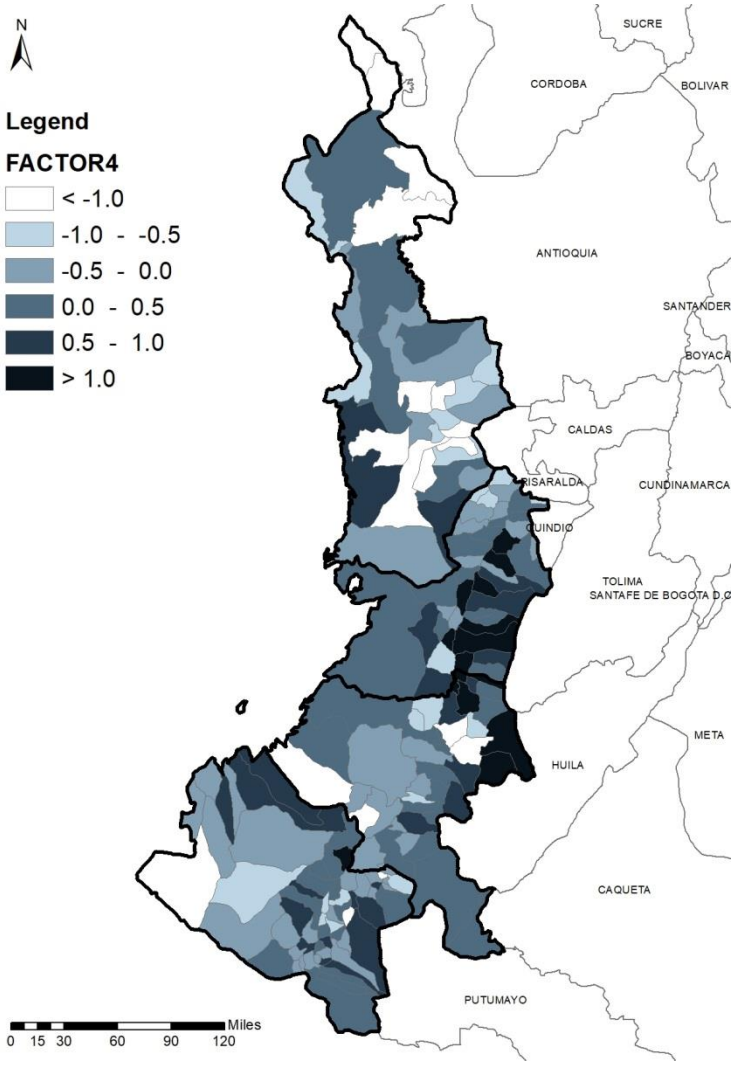


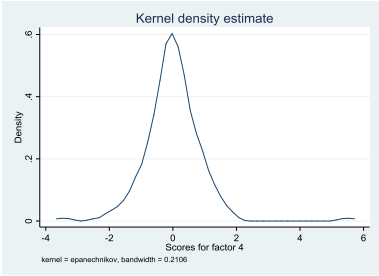
Factor 3 – Dimensión social



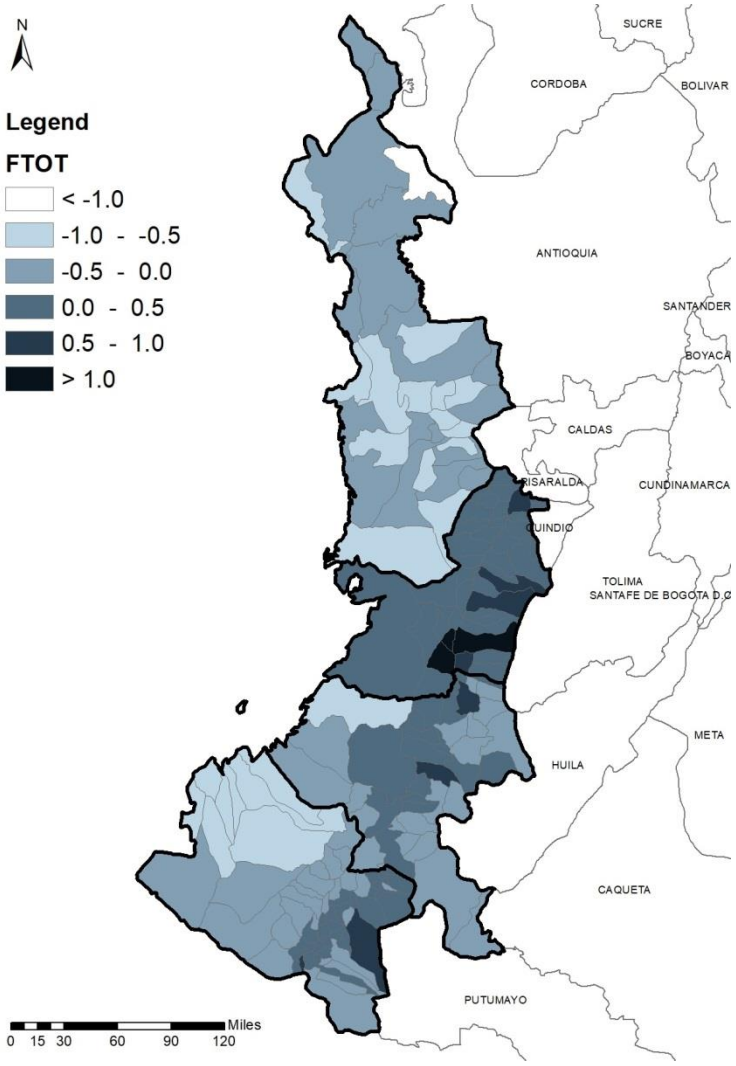


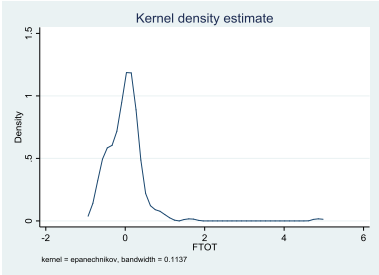
Factor 4 – Dimensión institucional





“Indicador de Potencial de Desarrollo Municipal: Región Pacífico de Colombia”



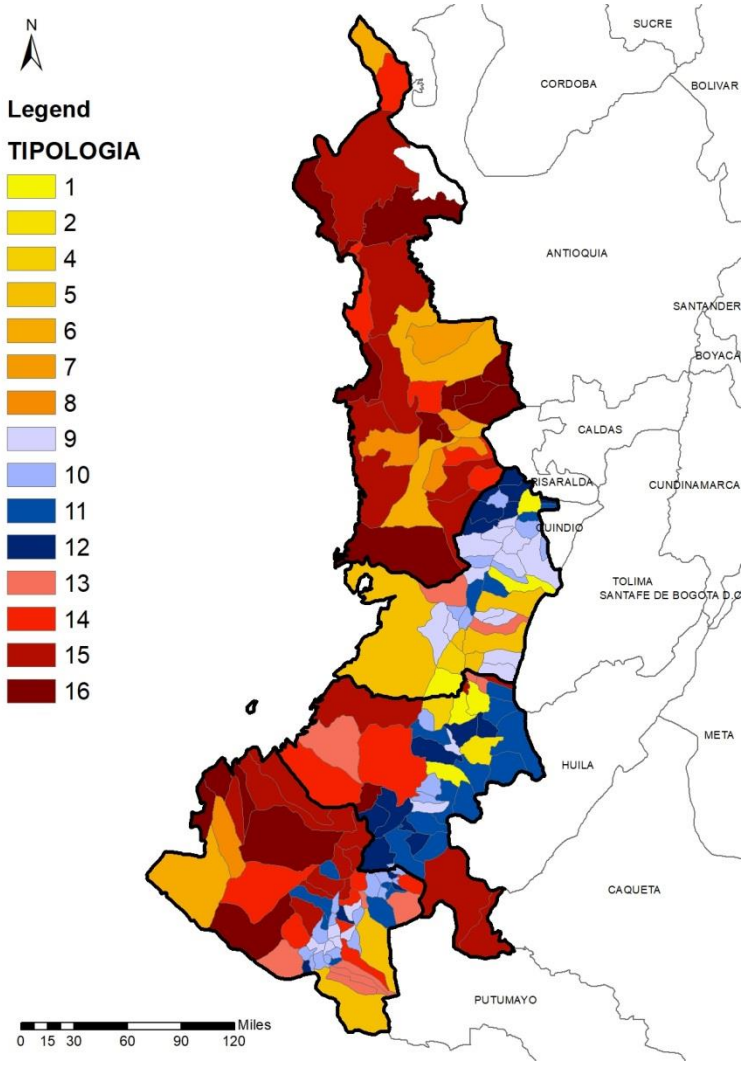


Tipología

FaCtor 2 – "AMBIENTAL"	Factor 1 – "ECONÓMICA" (+) – Factor 3 – "SOCIAL" (+)	Factor 1 – "ECONÓMICA" (+) – Factor 3 – "SOCIAL" (-)	Factor 1 – "ECONÓMICA" (-) – Factor 3 – "SOCIAL" (+)	Factor 1 – "ECONÓMICA" (-) – Factor 3 – "SOCIAL" (-)
(+)	Caloto, Popayán, Tuluá, Cartago, Santander de Quilichao, Jamundí Silvia	Cali, Buenos Aires	Bugalagrande, Zarzal, Roldanillo, Ginebra, Riofrío, Imués, Pradera, La Cumbre, Sevilla, Guacarí, La Unión, Guaitarilla, Dagua, La Florida, Bolívar, Florida, La Sierra, Sapuyes, Piendamó, Ospina, Yacuanquer Aldana, Caicedonia, Ancuyá, Trujillo, Gualmatán, La Unión, Belén, La Victoria, Argelia, Túquerres, Restrepo, Guachucal, San Pablo, Andalucía, Arboleda, Vijes, Iles, Rosas, Tangua, Suárez, Timbío, Pupiales, Florencia, San Lorenzo, Linares, Nariño	San Pedro, San Pedro de Cartago, Alcalá, Yotoco, Obando, Bolívar, Morales, Totoró, Buesaco, Albán, Inzá, Puracé, Contadero, Corinto, Toribio, Paez, Providencia, La Vega, Sotara, San Sebastián, Samaniego, Cumbitara Ulloa, Cuaspud, Colón, El Cairo, Versalles, Ansermanuevo, El Dovio, Toro, El Águila, Cajibío, Sandoná, Patía, Caldono, Sucre, Almaguer, San Bernardo, Balboa, Mercaderes, Jambaló
(-)	Yumbo, Palmira, Pasto, Guadalajara de Buga, Candelaria, Buenaventura, Ipiales Quibdó, San Andres de Tumaco, Istmina, Acandí, Tadó	Medio Atrato Roberto Payán, Medio Baudó, Cértegui, Medio San Juan, Río Iro	El Cerrito, Calima, El Tablón de Gómez, Chachagüí, Córdoba, Puerto Tejada, Potosí, Puerres, Cumbal, Padilla, Timbiquí El Tambo, Consaca, La Cruz, El Tambo, Taminango, Funes, Unguía, Mallama, San José del Palmar, Guapi, Bahía Solano, Condoto, Barbacoas, Río Quito	Miranda, Villa Rica, Guachené, El Rosario, Leiva, Policarpa, El Peñol, La Llanada, Riosucio, Bajo Baudó, Piamonte, Los Andes, Bojaya, Santa Rosa, Nóvita, Santacruz, López, Santa Bárbara, El Charco, Sipí, Olaya Herrera, Alto Baudo Argelia, Ricaurte, El Carmen de Atrato, Unión Panamericana, El Cantón del San Pablo, Bagadó, Carmen del Darien, Nuquí, Lloró, Juradó, La Tola, Francisco Pizarro, Mosquera, Magüi, Atrato, El Litoral del San Juan

OBS. Factor 4 – "INSTITUCIONAL" - encima de la media / abajo de la media

Tipología


N
Legend
TIPOLOGIA
1
2
4
5
6
7
8
9
10
11
12
13
14
15
16
SUCRE
CORDOBA
BOLIVAR
ANTIOQUIA
SANTANDER
BOYACA
CALDAS
RISARALDA
CUNDINAMARCA
QUINDIO
TOLIMA
SANTAFE DE BOGOTA D.C
META
HUILA
CAQUETA
PUTUMAYO
Miles
0 15 30 60 90 120

Reference

The notes for this lecture were adapted from previous material prepared by Prof. Eduardo Haddad for the course "Regional and Urban Economics", held yearly at the Department of Economics at the University of Sao Paulo.

They also relied on notes prepared by Oscar Torres-Reyna for the session on "Getting Started in Factor Analysis (using Stata 10)", available at:

https://dss.princeton.edu/training/Factor.pdf

We thank Dr. Inácio Araújo for the excellent assistance in preparing the material for this part of the course.