NEREUS

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Lecture 5: Input-Output Models

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Input-output analysis

Idea developed by Wassily Leontief (awarded Nobel Prize in Economics in 1973)

Employed in all countries – no matter what their political sentiments

Part of **N**ational **I**ncome and **P**roduct **A**ccounts

Extend ideas of the economic base model by disaggregating production into a set of sectors

Can be extended to explore issues of income distribution, tax policy, development strategies etc.

Input-output analysis

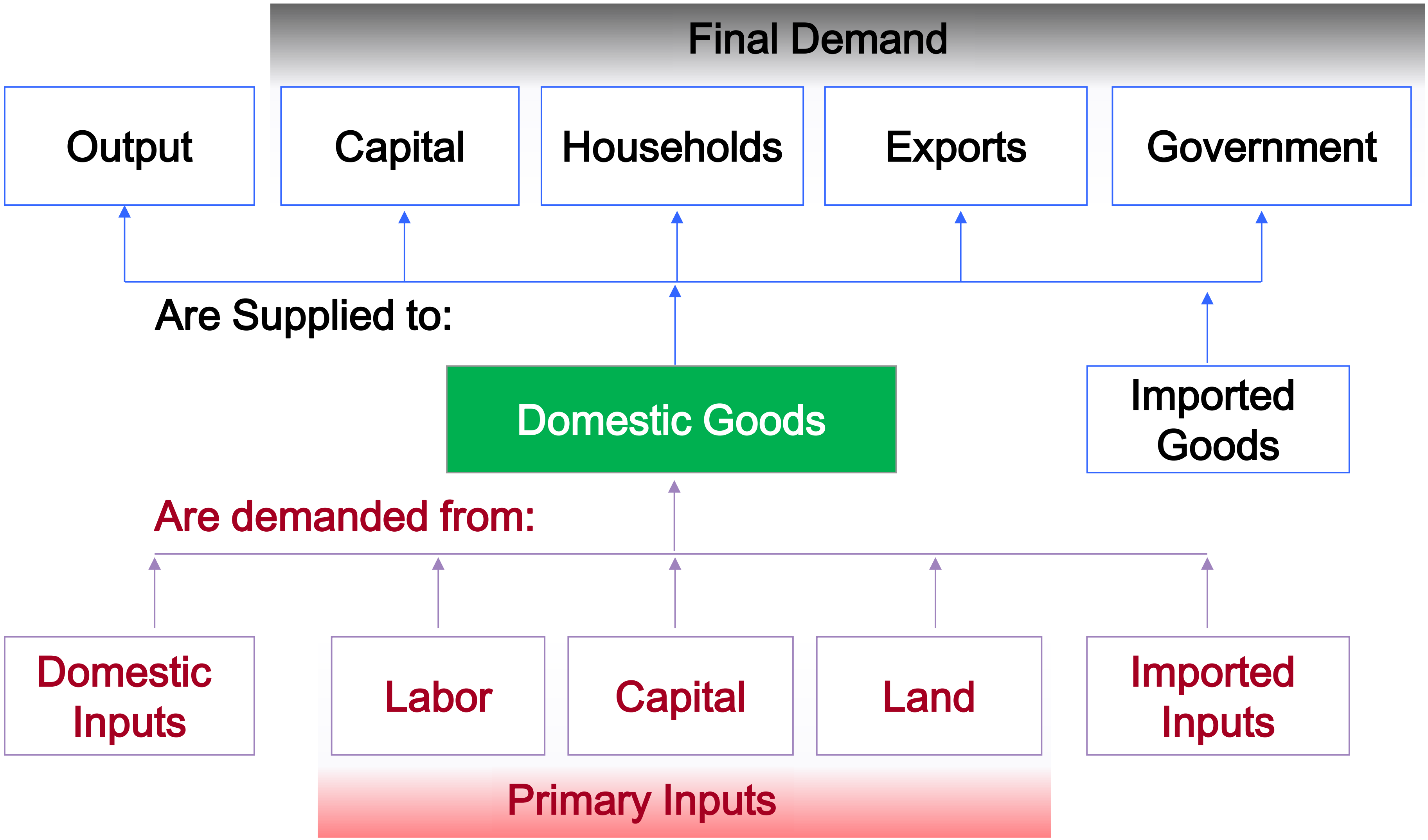
Imagine a region with *m* firms, producing a whole array of goods and services from agriculture, food processing, manufacturing, personal and business services and government.

Firms are assigned to *n* broad sectors based on their principal product.

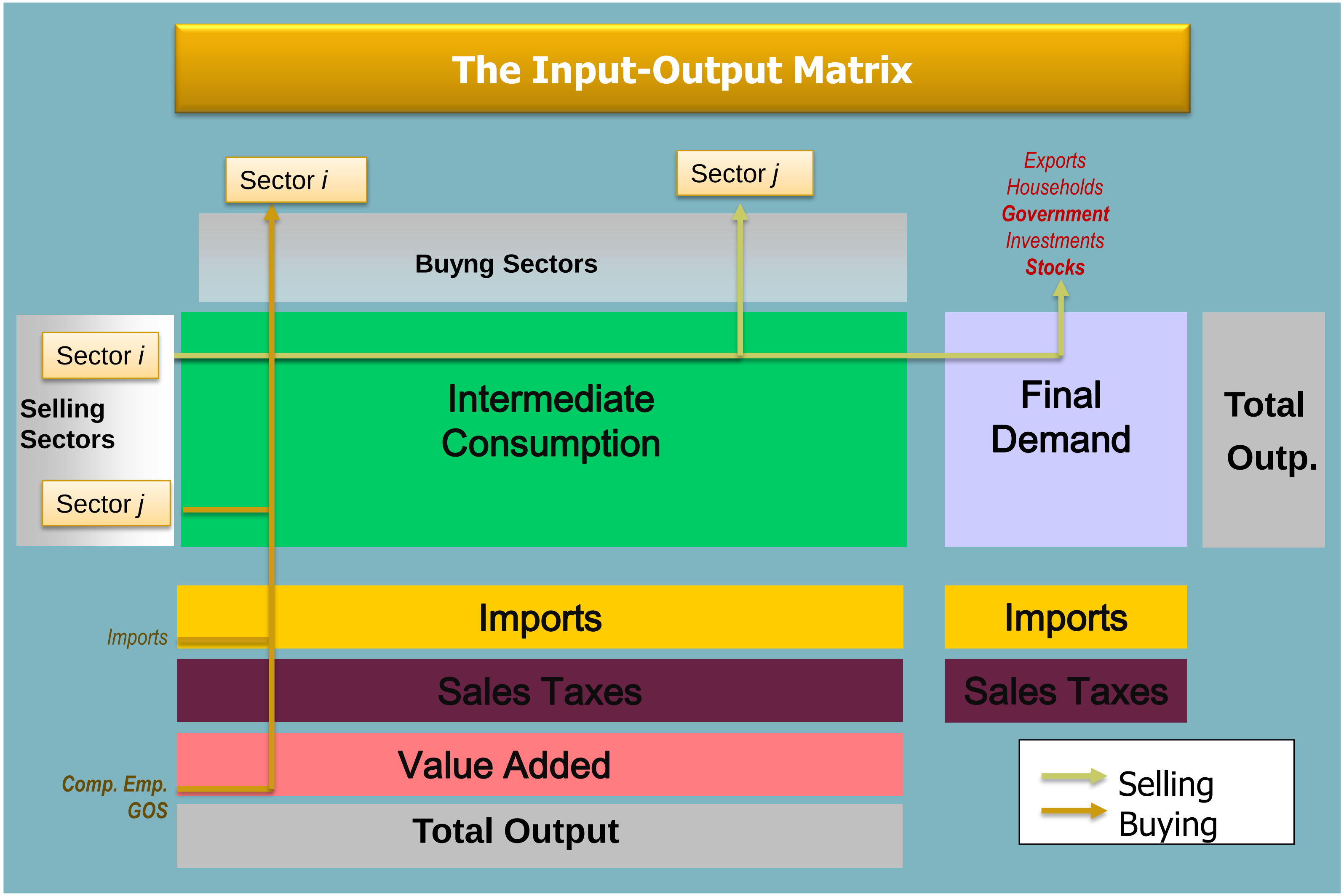
The number of sectors, *n*, ranges from 20 to several hundred and the allocation conforms to the Standard Industrial Classification (SIC).

For this presentation, only two sectors will be shown to facilitate the analysis and to avoid getting bogged down in details.

Input-output flows



Input-output table



Numerical example

IO Matrix	S1	S2	Y	X
S1	150	500	350	1000
S2	200	100	1700	2000
W	650	1400		
X	1000	2000		
Employment	300	800		

(First example in the Excel file)

Input-output flows

The transactions between these sectors are arrayed in a matrix (*n* rows and *n* columns), as shown in the table.

Looking across the *rows*, the sales made by the firm at the left can be traced to firms listed at the top of the column.

Thus sector 2 sells $200 to sector 1, and $100 to sector 2.

The *columns* provide complementary information of the source of purchases made by the sector at the top of the column from all other sectors.

Again, following sector 2, note that it buys $500 from sector 1, and $100 from sector 2.

This part of the input-output table is referred to as the *interindustry transactions*; it provides an *economic photograph* of the ways in which one sector is linked to another sector.

Input-output flows

However, sectors also make sales to other sets of activities – consumers, government and to customers located outside the region (exports).

In addition, firms also make purchases of labor (wages and salaries), returns to capital (profits and dividends) and imports.

These are shown in the rest of the table.

The column Y is referred to as *final demand*; row W is referred to as primary inputs.

The sum of wages and salaries and profits and dividends (returns to labor and capital) are referred to as *value added*.

Input-output model

The input-output table is basically an accounting system – a double entry one similar to that prepared for a business in which sales and purchases or assets and liabilities will be shown but, in this case, for an economy.

The next step is to **prepare an economic model** so that we can trace the impact of changes in one sector on the rest of the economy.

We do this because the nature of interdependence among sectors varies.

Input-output model: key assumptions

We assume that each sector produces goods and services according to a fixed recipe (formally known as a **production function**)

$$a_{ij} = \frac{z_{ij}}{x_j} \quad , \quad i, j = 1, \ldots, n$$

- Fixed technical coefficient
- Constant returns to scale
- Sectors use inputs in fixed proportions

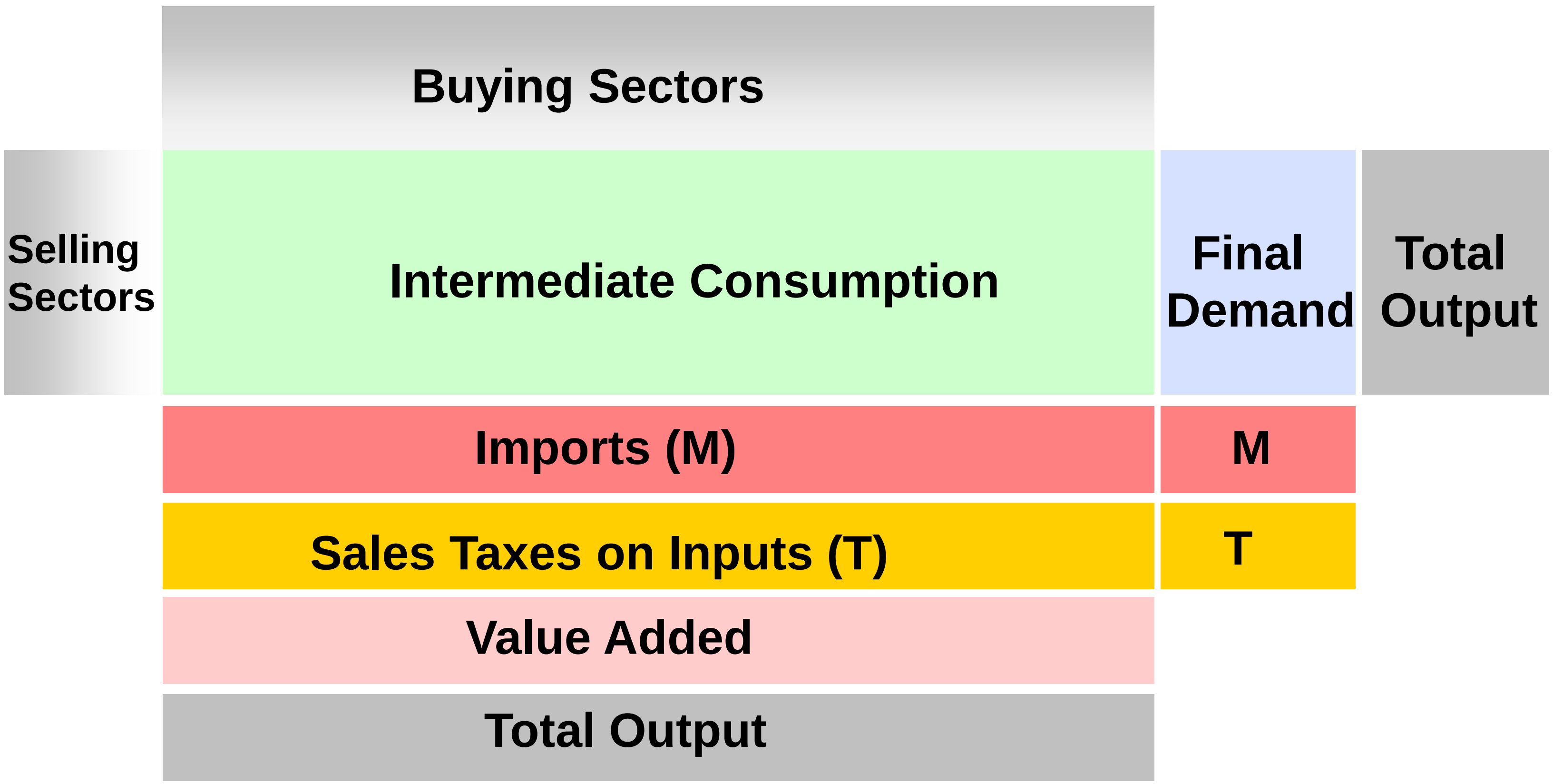
Input-output model: key assumptions

Inputs are expressed in monetary terms since it would be difficult to combine tons of iron ore with megawatts of electricity, or hours of labor in some consistent fashion.

This fixed recipe enables us to express the transactions in proportional form, also known as *direct coefficients*; these are shown in **the first example in the Excel file**.

The final assumption is that the economy is driven by signals emanating from final demand (consumers, government, exports); this is the *exogenous* part of the economy, while the interindustry transactions respond to these signals and are therefore *endogenous*.

Basic relations



Basic relations

$$\sum_{j=1}^{n} z_{ij} + y_i \equiv x_i \quad , \quad i = 1,\ldots,n$$

$$a_{ij} = \frac{z_{ij}}{x_j} \quad , \quad i, j = 1,\ldots,n$$

Production function

$$x_j = f\left(z_{1j},\ldots,z_{nj},W_j,M_j\right)$$

$$x_j = \min\left(\frac{z_{1j}}{a_{1j}},\ldots,\frac{z_{nj}}{a_{nj}}\right)$$

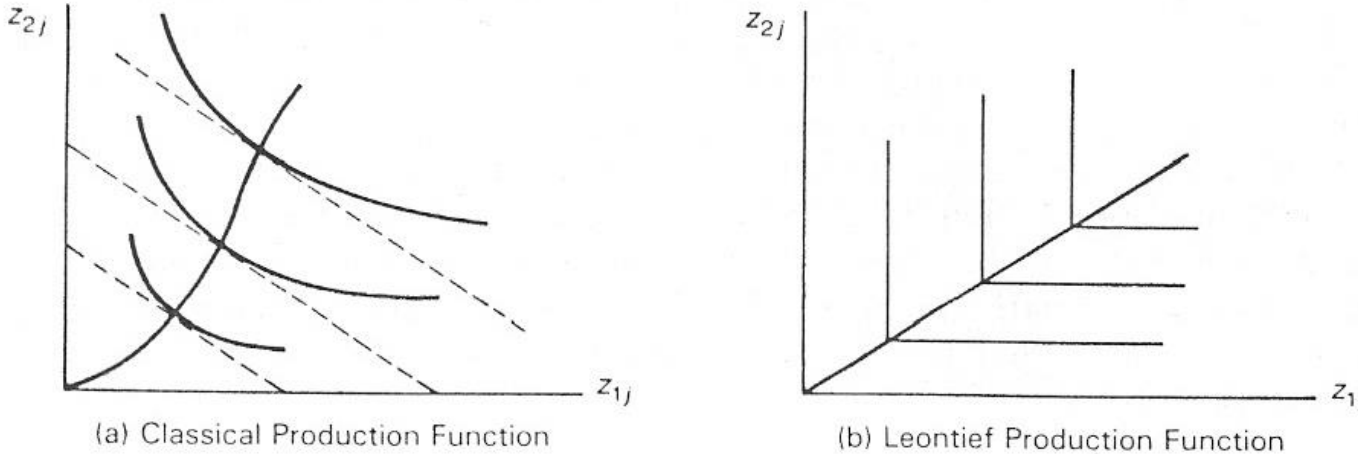


FIGURE 2-1 Production functions in input space.

The Leontief model

$$\sum_{j=1}^{n} a_{ij} x_j + y_i = x_i \qquad i = 1,\ldots,n$$

$$Ax + y = x$$

$$x = (I - A)^{-1} y$$

$$B = (I - A)^{-1}$$

The power series approximation of $(I - A)^{-1}$

$$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots)$$

Leontief matrix

$(I-A)^{-1}$ is known as the Leontief inverse matrix and is shown in the table below:

The entries reveal the direct and indirect impacts on a sector when final demand in the sector at the top of the column changes by $1 (or $1 million or $100 million).

Note that the entry on the principal diagonal is always >1; the unit value represents the increase in final demand in that sector. The remaining portion is the direct and indirect impact of expansion.

$(I-A)^{-1}$	1	2
1	1,254	0,330
2	0,264	1,122
Total	1,518	1,452

Leontief matrix

At the bottom of the multiplier table there is a row labeled **total**

Note that these values vary from 1.45 (sector 2) to 1.52 (sector 1)

How should these entries be interpreted?

They provide information on the impact on the rest of the economy (including the sector in question) of a unit change in final demand in any sector.

The value 1.45 for sector 2 tells us that for every increase of $1 in that sector an additional 0.45 worth of activity is generated for a total value of production of 1.45.

Leontief matrix

Why do these values vary?

- They reflect the degree to which a sector is dependent on other sectors in the economy for its inputs and as a source of consumption for its products.
- They depend on the structure of production (the recipe).

It would be incorrect to assume that a sector's importance in the economy is directly related to the size of the multiplier

- While true in part, a sector with a large volume of production but a modest multiplier may generate a greater volume of activity in the region than the sector with the largest multiplier but a smaller volume of production.

Multipliers and generators

There are several additional multipliers that can be calculated

When a sector expands production, it will increase payments to labor generating additional wages and salaries that will be spent in the region. Further, other industries whose production has to expand to meet these new demands will also spend more on wages and salaries. Thus, we may generate an **income multiplier** that reveals the relationship between direct income generation and total income (in similar fashion to output).

We could also transform the analysis into **employment** terms.

Multipliers and generators

Coefficient $$C_j^v = \frac{V_j}{X_j}$$

Generator
$$G_j^v = \sum_{i=1}^{n} c_i^v b_{ij}$$
$$G^v = C^v B \rightarrow 1xn$$
or
$$G^v = \hat{C}^v B \rightarrow nxn$$

Multiplier $$M_j^v = \frac{G_j^v}{C_j^V}$$

Exercise 1 – National IO

Data: Moroccan national input-output table, 2012

Calculate:

1. The sectoral output multipliers
2. The value added generator for each sector
3. The sectoral value added multipliers

Impact analysis

$$X = (I - A)^{-1} Y$$

$$\Delta X = (I - A)^{-1} \Delta Y$$

$$\Delta V = \hat{C}^{v} \Delta X$$

$$\Delta V = \hat{C}^{v} B \Delta Y$$

$$G^{v} = \hat{C}^{v} B$$

Exercise 2 – National IO

Data: Moroccan national input-output table, 2012

Calculate the impact on sectoral gross output and value added of the following scenarios:

1. An increase of 6.25% of exports of mining products
2. An increase of 6.25% of exports of food industry products
3. An increase of 0.18% of household consumption
4. An increase of investments equivalent to 1,000 millions DHS

Closing the IO model to households

Households earn incomes (at least in part) in payment for their labor inputs to production processes, and, as consumers, spend their income in rather well patterned ways.

One could move the household sector from the final-demand column and labor-input row and place it inside the technically interrelated table, making it one of the ***endogenous*** sectors.

$$\Delta X \rightarrow \Delta W \rightarrow \Delta Y$$

(Second example in the Excel file)

Closing the IO model to households

$$\overline{A} = \begin{bmatrix} A & H_c \\ H_r & 0 \end{bmatrix} \qquad \overline{Y} = \begin{bmatrix} Y^* \\ Y^*_{n+1} \end{bmatrix} \qquad \overline{X} = \begin{bmatrix} X \\ X_{n+1} \end{bmatrix}$$

$$Y^* = Y - Y^h$$

$$\overline{X} = \overline{B}\,\overline{Y}$$

$$\overline{B} = (I - \overline{A})^{-1}$$

Regional IO models

	Buying Sectors		
Selling Sectors	Intermediate Consumption	Final Demand	Total Output
	Imports from the rest of the Country (MC)	MC	
	Imports from the rest of the World (MW)	MW	
	Sales Taxes on Inputs (T)	T	
	Value Added		
	Total Output		

Interregional IO models

	Buying Sectors Region L	Buying Sectors Region M			
Selling sectors **Region L**	Interindustry Inputs LL	Interindustry Inputs LM	FD LL	FD LM	TO L
Selling sectors **Region M**	Interindustry Inputs ML	Interindustry Inputs MM	FD ML	FD MM	TO M
	Imports from the World	Imports from the World	M	M	M
	Sales Taxes	Sales Taxes	T	T	T
	Value Added	Value Added			
	Total Output L	Total Output M			

Interregional IO models

Interregional IO models

Estimation of regional models

Early studies:

$$p_j^R = \frac{\left(X_j^R - E_j^R\right)}{\left(X_j^R - E_j^R + M_j^R\right)}$$

where:

X_j^R is the total output of good j in region R;

E_j^R is the total exports of good j from region R;

M_j^R is the total imports of good j by region R.

$$A^R = \hat{P}A \longrightarrow X^R = \left(I - \hat{P}A\right)^{-1} Y^R$$

Estimation of regional models

Example:

$$\hat{P} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix}$$

$$A^R = \hat{P}A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix} \begin{bmatrix} 0.15 & 0.25 \\ 0.20 & 0.05 \end{bmatrix} = \begin{bmatrix} 0.12 & 0.20 \\ 0.12 & 0.03 \end{bmatrix}$$

$$(I - A^R)^{-1} = \begin{bmatrix} 1.169 & 0.241 \\ 0.145 & 1.061 \end{bmatrix}$$

(Third example in the Excel file)

Regional coefficients

Regional coefficients:

$$a_{ij}^{LL} = \frac{z_{ij}^{LL}}{X_j^L}$$

Regional production:

$$X^L = \left(I - A^{LL}\right)^{-1} Y^L$$

Interregional IO models

Interregional flows – intermediate consumption:

$$Z = \begin{bmatrix} Z^{LL} & Z^{LM} \\ Z^{ML} & Z^{MM} \end{bmatrix}$$

Total output:

$$X_i = z_{i1} + z_{i2} + \ldots + z_{ii} + \ldots + z_{in} + Y_i$$

$$X_1^L = z_{11}^{LL} + z_{12}^{LL} + z_{11}^{LM} + z_{12}^{LM} + Y_1^L$$

Interregional IO models

Interregional coefficients:

$$a_{ij}^{LL} = \frac{z_{ij}^{LL}}{X_j^L} \qquad a_{ij}^{LM} = \frac{z_{ij}^{LM}}{X_j^M}$$

$$a_{ij}^{ML} = \frac{z_{ij}^{ML}}{X_j^L} \qquad a_{ij}^{MM} = \frac{z_{ij}^{MM}}{X_j^M}$$

Deriving the interregional IO model

$$A = \begin{bmatrix} A^{LL} & \vdots & A^{LM} \\ \cdots & \cdots & \cdots \\ A^{ML} & \vdots & A^{MM} \end{bmatrix} \qquad Y = \begin{bmatrix} Y^{L} \\ \cdots \\ Y^{M} \end{bmatrix} \qquad X = \begin{bmatrix} X^{L} \\ \cdots \\ X^{M} \end{bmatrix}$$

$$\left\{ \begin{bmatrix} I & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & I \end{bmatrix} - \begin{bmatrix} A^{LL} & \vdots & A^{LM} \\ \cdots & \cdots & \cdots \\ A^{ML} & \vdots & A^{MM} \end{bmatrix} \right\} \begin{bmatrix} X^{L} \\ \cdots \\ X^{M} \end{bmatrix} = \begin{bmatrix} Y^{L} \\ \cdots \\ Y^{M} \end{bmatrix}$$

$$(I - A)X = Y,$$

$$X = (I - A)^{-1} Y$$

(Fourth example in the Excel file)

Multipliers in (inter)regional IO models

Multipliers vary not only across sectors but also across regions.

A small regional economy, with a modest representation of industry, may not be able to provide all the necessary inputs required by local industry. Thus, there will be considerable importation of inputs (sometimes referred to as leakages).

In general, the larger the value of the imports, the lower the value of the multiplier.

We would expect multipliers to decrease as we move from the country as a whole to a macro-region, an individual province, a metropolitan region and finally to a municipality.

Moroccan interregional input-output system, 2013

Interregional Input-Output Table for Morocco, 2013

<u>Reference:</u> Haddad, E. A., El-Hattab, F. and Ait-Ali, A. (2017). A Practitioner's Guide for Building the Interregional Input-Output System for Morocco, 2013. OCP Policy Center Research Paper (RP-17/02).

List of sectors

Table A2. Industry Classification

Elements of Set Industries	Description
A00	Agriculture, forestry, hunting, related services
B05	Fishing, aquaculture
C00	Mining industry
D01	Food industry and tobacco
D02	Textile and leather industry
D03	Chemical and para-chemical industry
D04	Mechanical, metallurgical and electrical industry
D05	Other manufacturing, excluding petroleum refining
D06	Oil refining and other energy products
E00	Electricity and water
F45	Construction
G00	Trade
H55	Hotels and restaurants
I01	Transport
I02	Post and telecommunications
J00	Financial activities and insurance
K00	Real estate, renting and services to enterprises
L75	General public administration and social security
MNO	Education, health and social action
OP0	Other non-financial services

Polarization by Casablanca

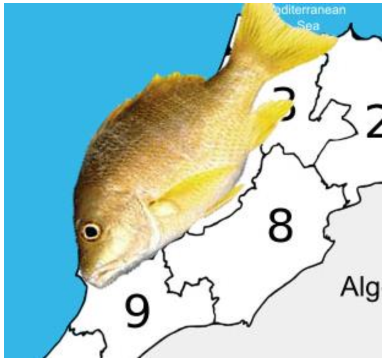
Casablanca region – ~30% of national GDP.

Asymmetries in the distribution of productive activity, with the primacy of Casablanca, serve to strengthen existing competitive advantages.

Presence of other relevant industrial areas outside Casablanca (the “fish”)

The “fish”

~ 80% of national GDP



Aggregate trade flows in Morocco

Table 2. Interregional Trade in Morocco, 2013 (in DHS millions)

		DESTINATION													TOTAL
		R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	EXP	
ORIGIN	R1	69,980	2,187	3,537	3,911	1,218	12,832	2,171	1,224	1,479	282	565	414	20,098	119,898
	R2	2,681	54,731	3,813	2,401	995	7,686	1,414	1,335	1,068	225	583	409	8,447	85,785
	R3	5,956	5,656	81,361	6,755	2,346	18,203	2,860	2,768	1,956	408	904	636	9,157	138,967
	R4	7,778	3,291	6,460	111,369	2,753	37,929	4,284	2,065	2,505	625	1,117	686	17,403	198,266
	R5	2,018	1,189	2,394	2,673	39,855	18,271	4,089	997	2,038	270	537	359	15,242	89,932
	R6	34,753	18,362	27,080	52,858	19,104	215,240	35,012	10,375	16,944	3,220	4,212	2,862	120,080	560,102
	R7	3,899	2,319	3,308	5,759	4,330	25,670	85,581	1,774	4,588	832	1,360	839	10,513	150,771
	R8	1,056	810	1,655	1,131	693	4,768	1,080	23,678	835	113	287	198	1,466	37,769
	R9	2,974	2,088	2,540	3,767	2,187	12,059	5,128	1,421	55,014	1,923	1,732	983	5,838	97,655
	R10	295	175	257	376	188	1,094	430	131	943	10,547	342	152	1,742	16,670
	R11	438	269	365	437	209	2,729	497	179	540	201	14,457	314	2,847	23,483
	R12	80	63	89	79	43	236	80	38	96	23	90	3,730	2,609	7,257
	IMP	48,842	26,748	37,534	47,529	21,206	160,187	41,340	12,627	23,625	3,651	5,759	2,579	0	431,626
TOTAL		180,749	117,887	170,393	239,045	95,127	516,904	183,965	58,612	111,628	22,321	31,946	14,161	215,444	1,958,182

Source: Calculations by the authors.

81.6% of total domestic flows

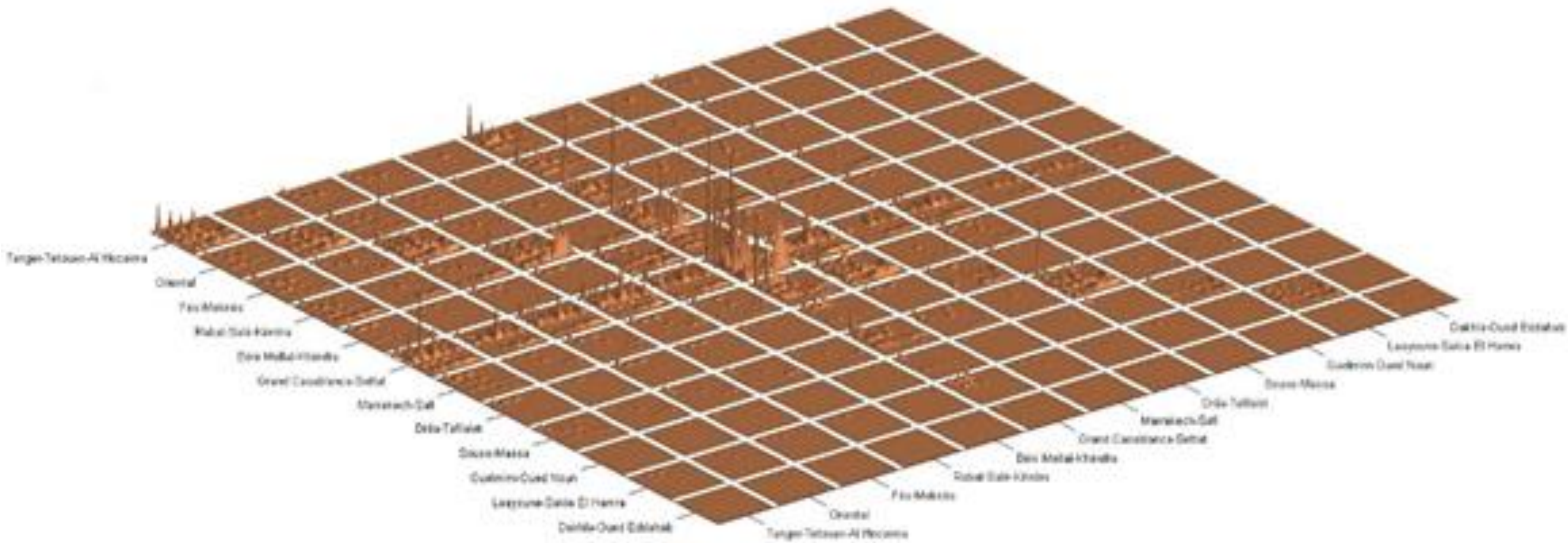
Sectoral Composition of the Total Production Value in the Regions of Morocco, 2013

		R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12
A00	Agriculture, forêt et services annexes	9.9	16.6	21.7	10.6	25.6	4.1	14.9	27.5	12.2	11.5	0.0	0.0
B05	Pêche, aquaculture	0.9	0.3	0.0	0.1	0.0	0.1	0.3	0.0	4.2	9.3	6.4	38.6
C00	Industrie d'extraction	0.0	1.3	0.1	0.4	21.9	0.0	6.9	9.0	0.0	0.0	12.2	0.0
D01	Industries alimentaires et tabac	6.3	3.3	11.5	4.6	6.5	15.6	6.2	1.7	21.1	6.7	10.6	13.0
D02	Industries du textile et du cuir	8.6	0.5	4.4	2.8	0.0	5.4	0.8	0.0	0.0	0.0	0.0	0.0
D03	Industrie chimique et parachimique	1.2	0.6	1.3	1.5	0.2	7.9	3.5	0.1	0.9	0.0	3.8	0.0
D04	Industrie mécanique, métallurgique et électrique	21.0	5.4	3.4	4.6	0.7	10.6	0.4	0.1	1.2	0.0	0.2	0.0
D05	Autres industries manufac. hors raffinage pétro	5.5	1.6	3.0	2.3	1.0	8.3	2.8	0.3	3.2	0.9	3.4	1.6
D06	Raffinage de pétrole et autres produits d'énergie	0.0	0.0	0.0	0.0	0.0	8.8	0.0	0.0	0.0	0.0	0.0	0.0
E00	Electricité et eau	2.7	2.6	2.5	3.4	1.9	1.3	2.4	2.0	2.3	1.9	1.8	1.2
F45	Bâtiment et travaux publics	11.2	13.9	7.9	8.0	9.4	4.6	12.2	19.3	7.6	7.0	12.1	3.9
G00	Commerce	8.4	14.8	9.8	7.9	8.3	5.4	9.1	6.8	8.5	9.2	5.4	4.9
H55	Hôtels et restaurants	1.7	1.0	1.5	0.7	0.3	0.7	7.9	2.3	8.8	0.4	0.3	1.0
I01	Transports	4.0	6.4	4.9	4.8	3.5	3.4	4.3	4.5	4.6	6.2	3.4	3.3
I02	Postes et télécommunications	2.1	3.5	2.7	2.6	1.9	1.8	2.3	2.4	2.5	3.3	1.9	1.8
J00	Activités financières et assurances	2.3	3.7	2.8	6.7	1.9	5.0	3.5	2.2	3.1	2.1	1.4	1.3
K00	Immobilier, location et serv. rendus entreprises	4.1	6.5	5.0	12.0	3.4	8.9	6.3	3.9	5.5	3.8	2.4	2.3
L75	Administration publique et sécurité sociale	4.3	8.6	8.1	17.9	6.8	3.2	7.2	10.3	5.4	28.0	27.7	20.7
MNO	Education, santé et action sociale	4.5	8.1	8.3	7.2	5.9	4.1	7.2	6.7	7.7	8.8	6.4	6.0
OP0	Autres services non financiers	1.1	1.2	1.1	1.8	0.9	0.9	1.6	0.8	1.1	0.8	0.5	0.5

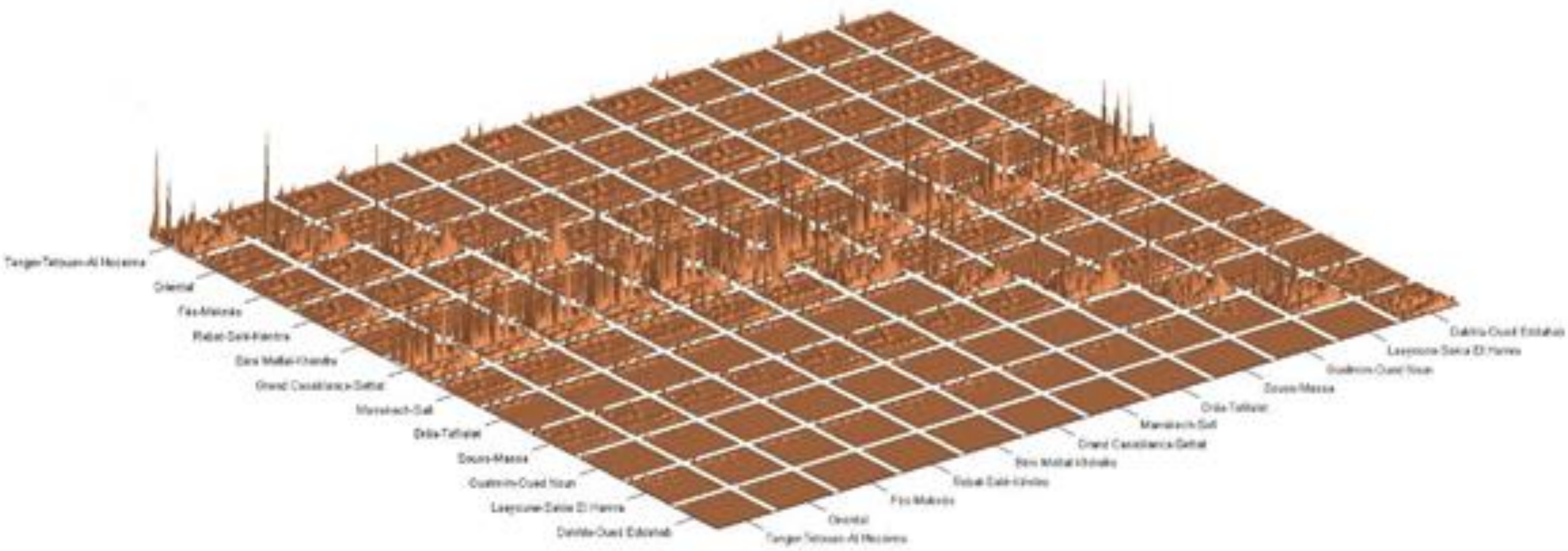
Regional Participation in the Total Production Value of Morocco, by sector, 2013

		R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12
A00	Agriculture, forêt et services annexes	7.8	8.3	17.8	12.3	13.4	13.0	13.1	6.1	7.0	1.1	0.0	0.0
B05	Pêche, aquaculture	9.8	2.1	0.0	1.2	0.0	4.2	3.8	0.0	32.8	12.2	11.9	22.1
C00	Industrie d'extraction	0.0	3.0	0.5	1.9	50.8	0.2	27.1	8.9	0.1	0.0	7.5	0.0
D01	Industries alimentaires et tabac	5.3	1.8	9.9	5.6	3.6	52.3	5.8	0.4	12.6	0.7	1.5	0.6
D02	Industries du textile et du cuir	21.3	0.8	11.3	10.1	0.0	54.0	2.2	0.0	0.1	0.0	0.0	0.0
D03	Industrie chimique et parachimique	2.7	1.0	3.1	5.1	0.3	75.3	9.3	0.1	1.5	0.0	1.6	0.0
D04	Industrie mécanique, métallurgique et électrique	26.4	4.3	4.4	8.4	0.5	54.1	0.6	0.0	1.1	0.0	0.0	0.0
D05	Autres industries manufac. hors raffinage pétro	10.3	1.9	5.9	6.4	1.2	62.5	5.8	0.1	4.4	0.2	1.1	0.2
D06	Raffinage de pétrole et autres produits d'énergie	0.0	0.0	0.0	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0	0.0
E00	Electricité et eau	11.4	6.9	11.0	20.8	5.3	21.9	11.0	2.3	7.0	1.0	1.3	0.3
F45	Bâtiment et travaux publics	12.1	9.6	8.9	12.8	6.7	20.1	14.7	5.9	5.9	0.9	2.3	0.2
G00	Commerce	9.6	10.7	11.7	13.3	6.2	25.0	11.6	2.2	7.0	1.3	1.1	0.3
H55	Hôtels et restaurants	7.1	2.8	6.4	4.1	0.9	12.0	36.8	2.8	26.4	0.2	0.2	0.2
I01	Transports	8.4	8.6	10.9	15.1	4.8	29.1	10.1	2.7	7.0	1.6	1.3	0.4
I02	Postes et télécommunications	8.4	8.6	10.9	15.1	4.8	29.1	10.1	2.7	7.0	1.6	1.3	0.4
J00	Activités financières et assurances	5.0	5.0	6.3	21.4	2.7	43.6	8.6	1.3	4.9	0.6	0.5	0.1
K00	Immobilier, location et serv. rendus entreprises	5.0	5.0	6.3	21.4	2.7	43.6	8.6	1.3	4.9	0.6	0.5	0.1
L75	Administration publique et sécurité sociale	5.0	6.4	9.8	30.5	5.2	15.1	9.3	3.4	4.5	4.0	5.6	1.3
MNO	Education, santé et action sociale	6.7	7.7	12.9	15.8	5.8	24.3	12.0	2.8	8.3	1.6	1.7	0.5
OP0	Autres services non financiers	8.6	6.2	8.8	21.1	4.7	26.9	14.3	1.8	6.0	0.7	0.7	0.2

Flows of Goods and Services for Production among the Regions of Morocco, 2013



Technical Dependence for Production (Leontief Inverse Matrix) among the Regions of Morocco, 2013



Interregional linkages

The conventional input-output model is given by the system of matrix equations:

$$x = (I - A)^{-1}f = Bf$$

where x and f are respectively the vectors of gross output and final demand; A consists of input coefficients a_{ij} defined as the amount of product i required per unit of product j (in monetary terms), for i, j = 1,…, n; and B is known as the Leontief inverse.

Interregional input-output model

In an interregional context, with *R* different regions, we have:

$x = \begin{bmatrix} x^1 \\ \vdots \\ x^R \end{bmatrix}$; $A = \begin{bmatrix} A^{11} & \cdots & A^{1R} \\ \vdots & \ddots & \vdots \\ A^{R1} & \cdots & A^{RR} \end{bmatrix}$; $f = \begin{bmatrix} f^1 \\ \vdots \\ f^R \end{bmatrix}$; and $B = \begin{bmatrix} B^{11} & \cdots & B^{1R} \\ \vdots & \ddots & \vdots \\ B^{R1} & \cdots & B^{RR} \end{bmatrix}$

and

$$
\begin{aligned}
x^1 &= B^{11} f^1 + \cdots + B^{1R} f^R \\
&\vdots \\
x^R &= B^{R1} f^1 + \cdots + B^{RR} f^R
\end{aligned}
$$

Interregional input-output model

Let us also consider different components of f, which include demands originating in the specific regions, v^{rs}, $s = 1,\ldots, R$, and abroad, e. We obtain information of final demand from origin s in the IIOM-MOR, allowing us to treat v as a matrix which provides the monetary values of final demand expenditures from the domestic regions in Morocco and from the foreign region.

$$v = \begin{bmatrix} v^{11} & \cdots & v^{1R} \\ \vdots & \ddots & \vdots \\ v^{R1} & \cdots & v^{RR} \end{bmatrix};\ e = \begin{bmatrix} e^{1} \\ \vdots \\ e^{R} \end{bmatrix}$$

Thus:

$$x^1 = B^{11}(v^{11} + \cdots + v^{R1} + e^1) + \cdots + B^{1R}(v^{1R} + \cdots + v^{RR} + e^R)$$
$$\vdots$$
$$x^R = B^{R1}(v^{11} + \cdots + v^{R1} + e^1) + \cdots + B^{RR}(v^{1R} + \cdots + v^{RR} + e^R)$$

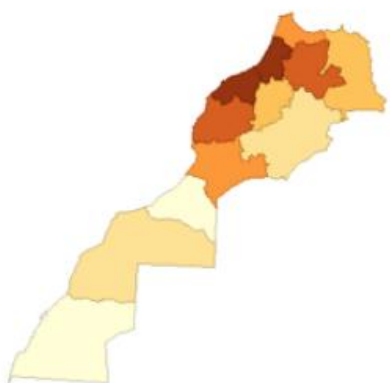
Spatial propagation of final demand shocks (…)

Tanger-Tetouan-Al Hoceima

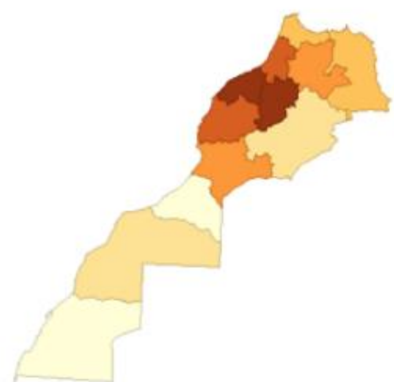
Oriental

Fès-Meknès

Rabat-Salé-Kénitra

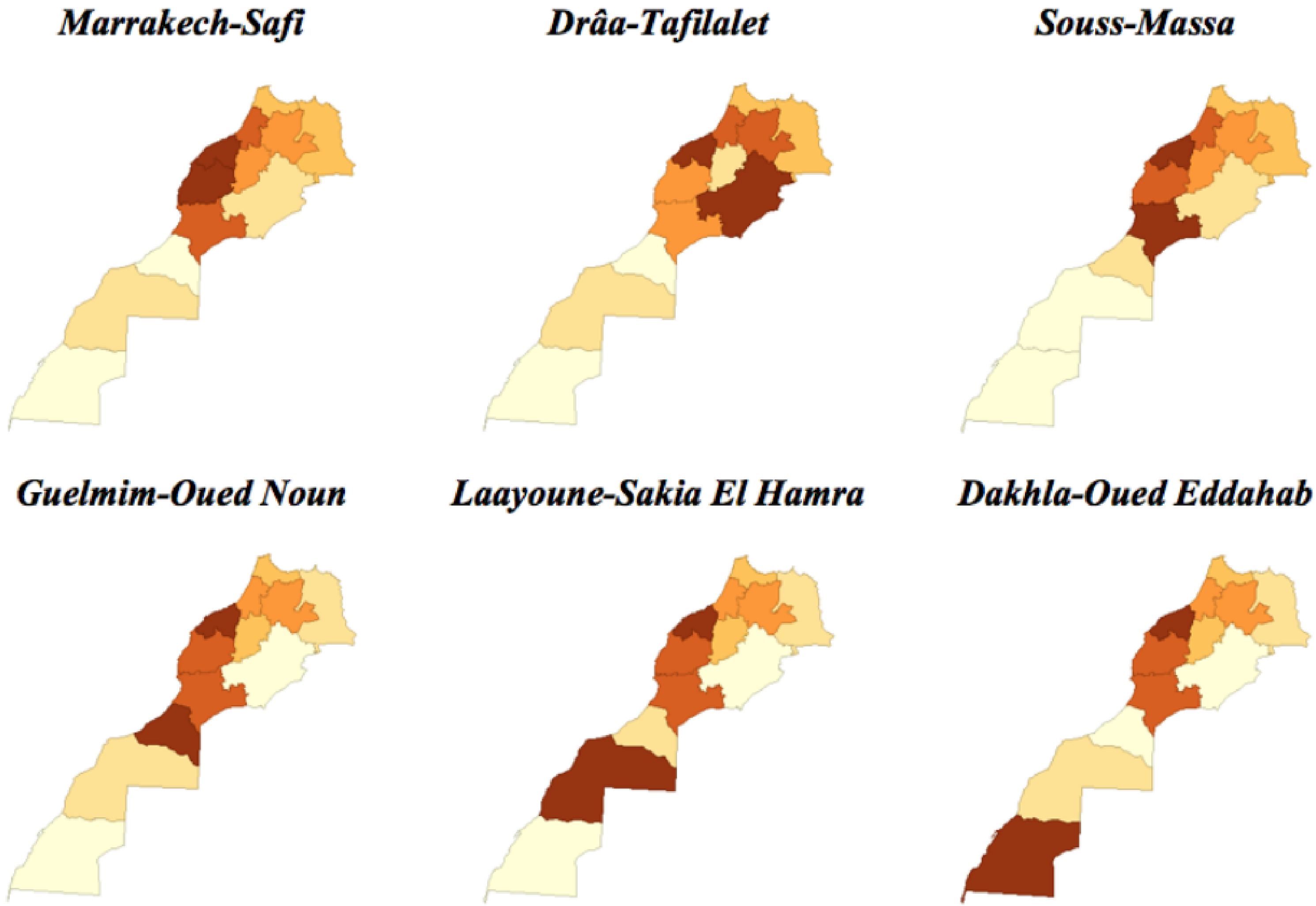


Béni Mellal-Khénifra



Grand Casablanca-Settat

Spatial propagation of final demand shocks



Many potential applications!

Input-output applications

- Moroccan regions (how do they relate?), structural decomposition analysis (historical estimation, updating), main drivers of sectoral and regional growth, impact of interregional government transfers, impact analysis…

Interregional CGE applications

- Economic impacts of drought, regional impacts of climate change (agriculture), specific transportation projects (accessibility), simulate TFP-enhancing policies (sectors and regions), other usual CGE applications, ...

Exercise 3 – Interregional IO (impact analysis)

Data: Moroccan two-region interregional input-output table, 2013

1. Calculate the share of Casablanca in total Moroccan foreign exports, by sector

2. Calculate total value added creation of foreign exports from Casablanca and from the rest of the country on:

    i. Casablanca
    ii. Rest of Morocco
    iii. Morocco

The Extraction Method

The extraction method, initially proposed by Dietzenbacher et al. (1993), consists of the hypothetical extraction of a sector/region in the input-output matrix

The purpose is to quantify how much the total output of an economy with *n* sectors could change (or reduce) if a particular sector/region were removed from this economy

This technique allows to analyzing the importance of a sector/region in an economic structure given its extraction and consequent reduction in the level of activity in the economy

It should be emphasized that the greater the level of interdependence of this sector/region in relation to the others, the greater the impact, in a systemic way

Example: sectoral extraction

Initially, the extraction was modeled by an input-product matrix deleting the *j-th* row and column of matrix ***A***

Using $\bar{\boldsymbol{A}}_{(j)}$ for the matrix of dimensions *(n - 1) x (n - 1)* without the sector *j* and $\bar{\boldsymbol{f}}_{(j)}$ for the reduced final demand vector (*i.e.* without sector *j*), production in the reduced economy (*i.e.* without sector *j*) will be given by:

$$\bar{\boldsymbol{x}}_{(j)} = \left(\boldsymbol{I} - \bar{\boldsymbol{A}}_{(j)}\right)^{-1} \bar{\boldsymbol{f}}_{(j)} \qquad (1)$$

Instead of physically deleting the *j-th* row and column in matrix ***A*** and the *j-th* element of vector ***f***, one can simply replace these values with zeros

Example: sectoral extraction

In the complete model, with *n* sectors, the output of the economy is given by:

$$\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{f} \quad (2)$$

Therefore, after extraction:

$$\boldsymbol{T}_j = \boldsymbol{i}'\boldsymbol{x} - \boldsymbol{i}'\overline{\boldsymbol{x}}_{(j)} \quad (3)$$

where $\boldsymbol{T}_j$ is the aggregate measure of loss in the economy – decrease in total output if the sector *j* "disappears". In other words, it is a measure of the relative importance of sector *j*, or the total linkages of sector *j*.

Exercise 4 – Interregional IO (extraction method)

Data: Moroccan two-region interregional input-output table, 2013

What are the gross output impacts of shutting down the Moroccan Refinery Samir?

Use the extraction method to answer the following questions:

1. What is the associated change in the sectoral output?
2. Which sectors potentially present higher relative losses?
3. How are the two regions in the model affected?