Lecture 8: CGE Models

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Outline

✓ Introduction

Structure of a CGE Model

The Johansen Approach

Stylized Johansen Model

Interregional CGE Models

What is a CGE?

Computable, based on data

It has many sectors

And perhaps many regions, primary factors and households

A big database of matrices

Many, simultaneous, equations (hard to solve)

Prices guide demands by agents

Prices determined by supply and demand

Trade focus: elastic foreign demand and supply

General features of CGE models

CGE models include equations specifying:

- market-clearing conditions for commodities and primary factors;
- producers' demands for produced inputs and primary factors;
- final demands (investment, household, export and government);
- the relationship of prices to supply costs and taxes;
- various macroeconomic variables and price indices.

Neo-classical flavor

- demand equations consistent with optimizing behavior (cost minimization, utility maximization);
- competitive markets: producers price at marginal cost.

CGE simplifications

Not much dynamics (leads and lags)

An imposed structure of behavior, based on theory

Neoclassical assumptions (optimizing, competition)

Nesting (separability assumptions)

Why: time series data for huge matrices cannot be found

Theory and assumptions (partially) replace econometrics

What is a CGE model good for?

Analyzing policies that affect different sectors in different ways

The effect of a policy on different:
- Sectors
- Regions
- Factors (Labor, Land, Capital)
- Household types

Policies (tariff or subsidies) that help one sector a lot, and harm all the rest a little

What-if questions

What if productivity in agriculture increased 1%?

What if foreign demand for exports increased 5%?

What if consumer tastes shifted towards imported food?

What if CO_2 emissions were taxed?

What if water became scarce?

A great number of exogenous variables (tax rates, endowments, technical coefficients)

Comparative static models: results show effect of policy shocks only, in terms of changes from initial equilibrium

Comparative-static interpretation of results

Results refer to changes at *some* future point in time
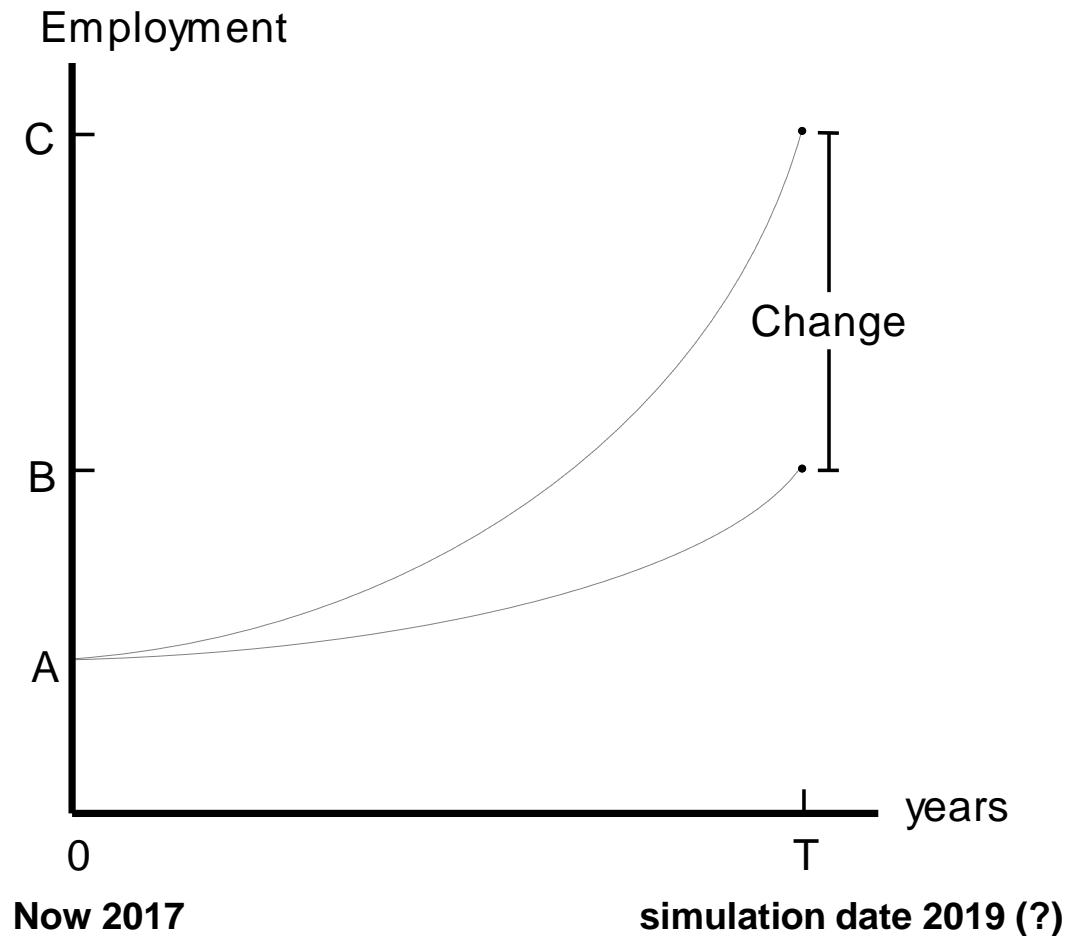
Regional modeling

Intense interest in regional results

Policies which are good for nation but bad for one region may not be politically feasible

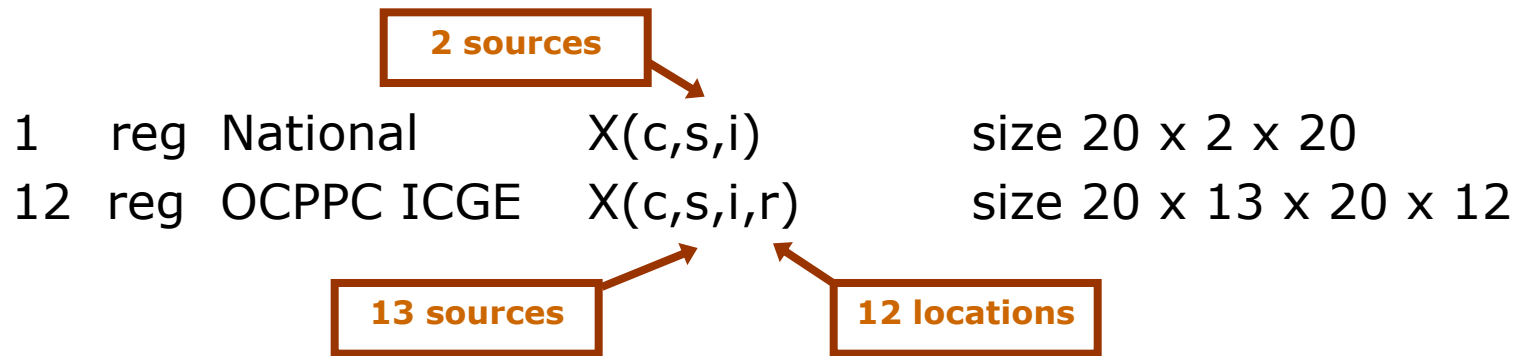Assistance to one region may harm nation

Regional modeling is the largest part of Fipe's Brazilian contract work

- More customers:
  1 national government but 27 state governments

Two approaches: **Bottom-Up** or Top-Down

The bottom-up approach

Simply add a regional subscript (or two) to each variable and data

2 sources

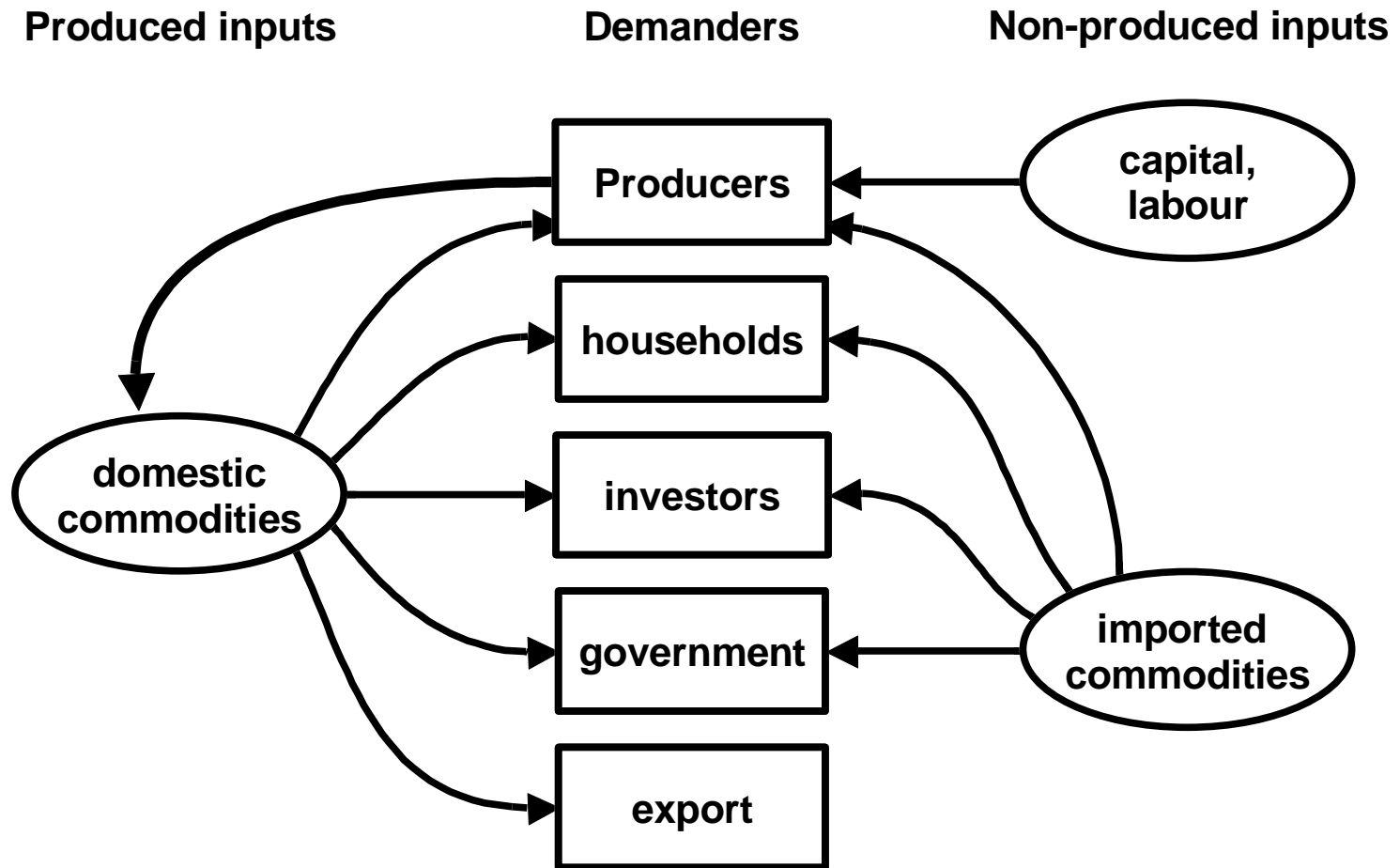| 1 | reg | National | $X(c,s,i)$ | size 20 x 2 x 20 |
| 12 | reg | OCPPC ICGE | $X(c,s,i,r)$ | size 20 x 13 x 20 x 12 |

13 sources   **12 locations**

Database has grown by factor of [13/2]*12 = 78

Number of variables also 78 times bigger

Solve time and memory needs move with SQUARE of model size
(so model needs 6,000 times as much memory and takes 6,000 times longer to solve)

Stylized GE model: material flows

You will learn…

How microeconomic theory – cost-minimizing, utility-maximizing – underlies the equations;

The use of nested production and utility functions:

How input-output data is used in equations;

How model equations are represented in percent change form;

How choice of exogenous variables makes model more flexible.

CGE models mostly similar, so skills will transfer.

Outline

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✓ **Structure of a CGE Model**
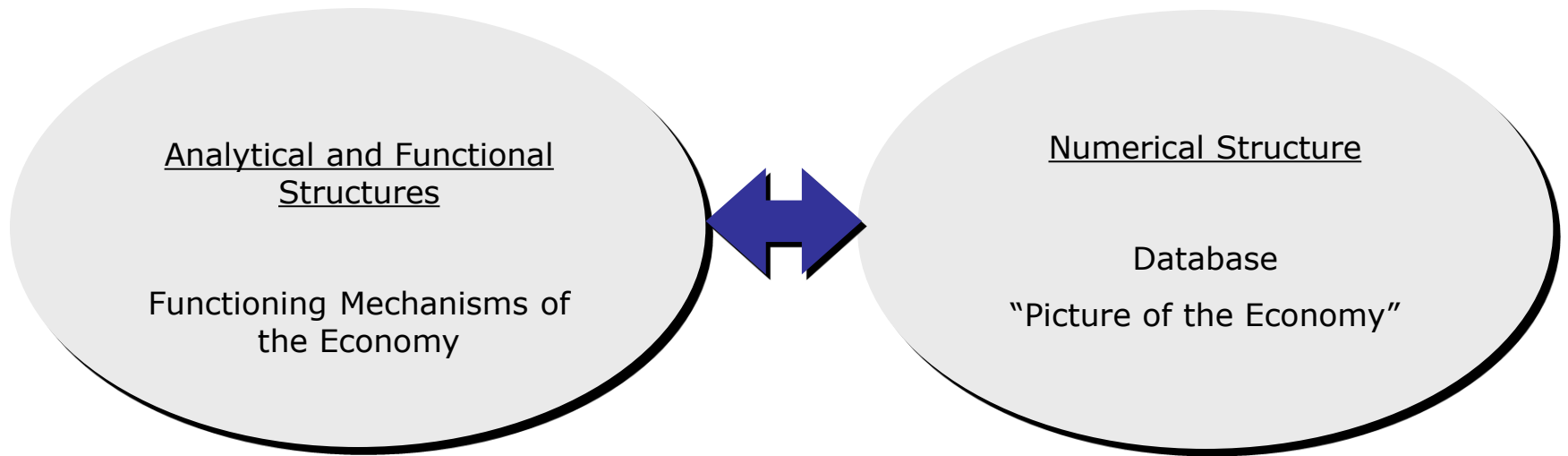
The Johansen Approach

Stylized Johansen Model

Interregional CGE Models

CGE models – Definition

Analytical and Functional Structures

Functioning Mechanisms of the Economy

Numerical Structure

Database

"Picture of the Economy"

Nested structures in CGE models

In each industry: Output = function of inputs:

output = F (inputs) = F (Labor, Capital, Land, dom. goods, imp. goods)

Separability assumptions simplify the production structure:

output = F (primary factor composite, composite goods)

where: primary factor composite = CES (Labor, Capital, Land)

                labor = CES (Various skill grades)

                composite good (i) = CES (domestic good (i), imported good (i))

All industries share common production structure.
BUT: Input proportions and behavioral parameters vary.

Nesting is like staged decisions:
First decide how much leather to use – based on output.
Then decide import/domestic proportions, depending on the relative prices of local and foreign leather.
Each nest requires 2 or 3 equations.

Inputs to production: Nests (example)

Activity Level

Leontief — top nest

Good 1 — Good G — Primary Factors — 'Other Costs'

CES — CES — CES

Domestic Good 1 — Imported Good 1 — Domestic Good G — Imported Good G — Land — Labour — Capital

Armington nest

primary factor nest

Work upwards

CES — **skill nest**

Labour type 1 — Labour type 2 — up to — Labour type O

KEY

Functional Form

Inputs or Outputs
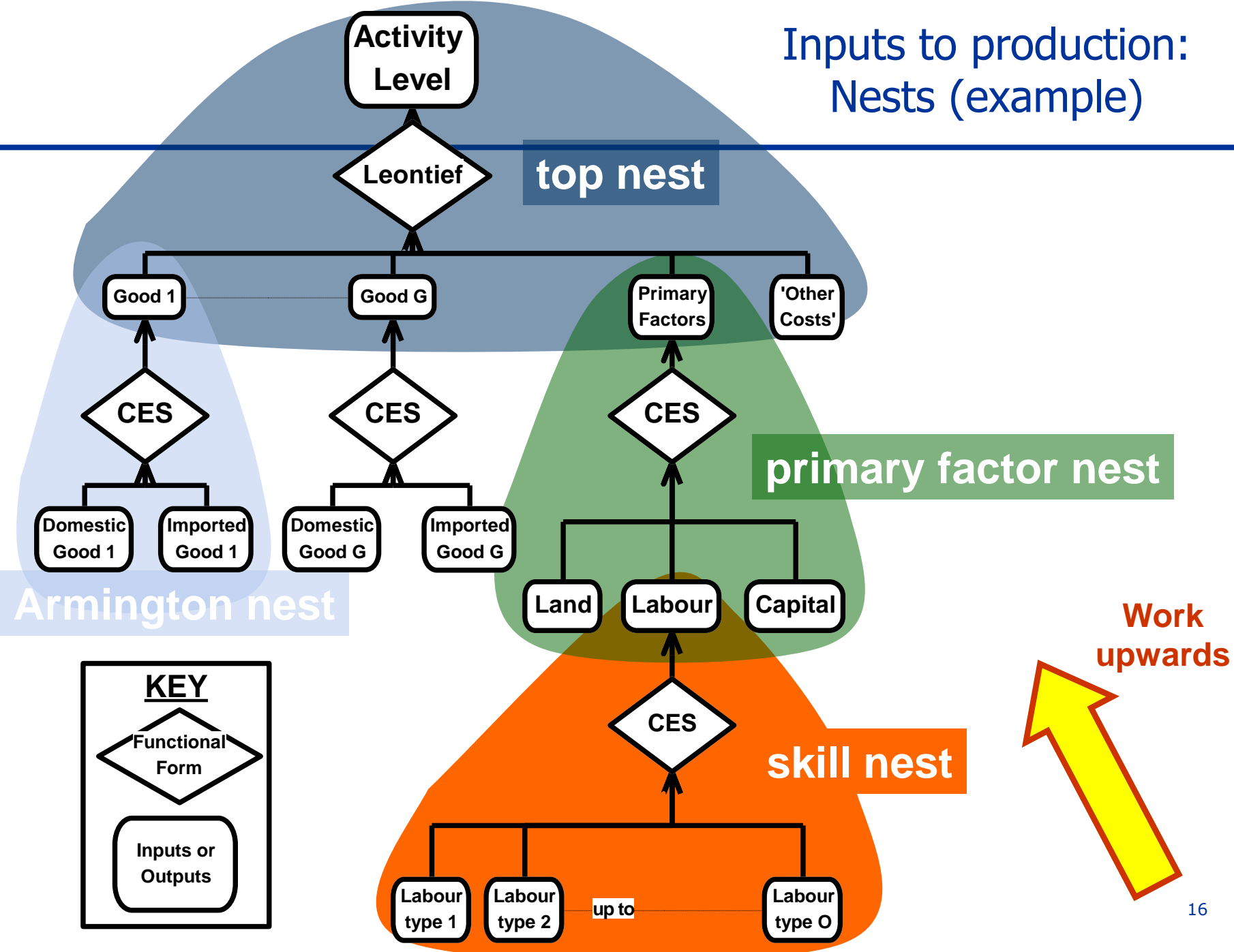
Example 1: CES skill substitution

Skilled
X_s

$$X^\alpha = X_s{}^\alpha + X_u{}^\alpha$$

$$0 < \alpha < 1$$

R

C

A

B

X=15

X=10

Cost=$6

Cost=$9

UnSkilled
X_u

Effect of price change

Unskilled wages fall relative to skilled wages

Skilled X$_s$

PR1

PR2

X=10

A

B

Unskilled X$_u$
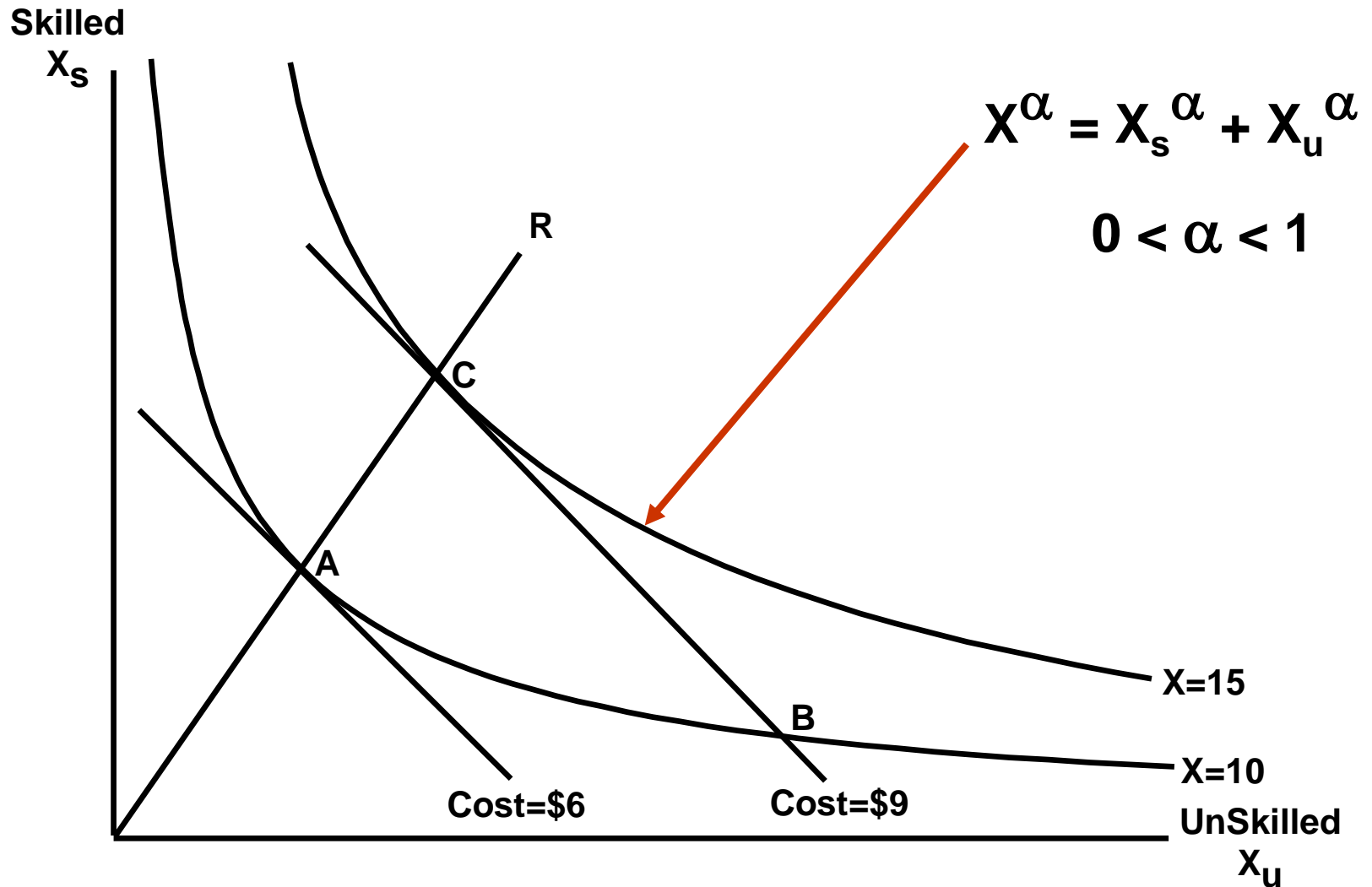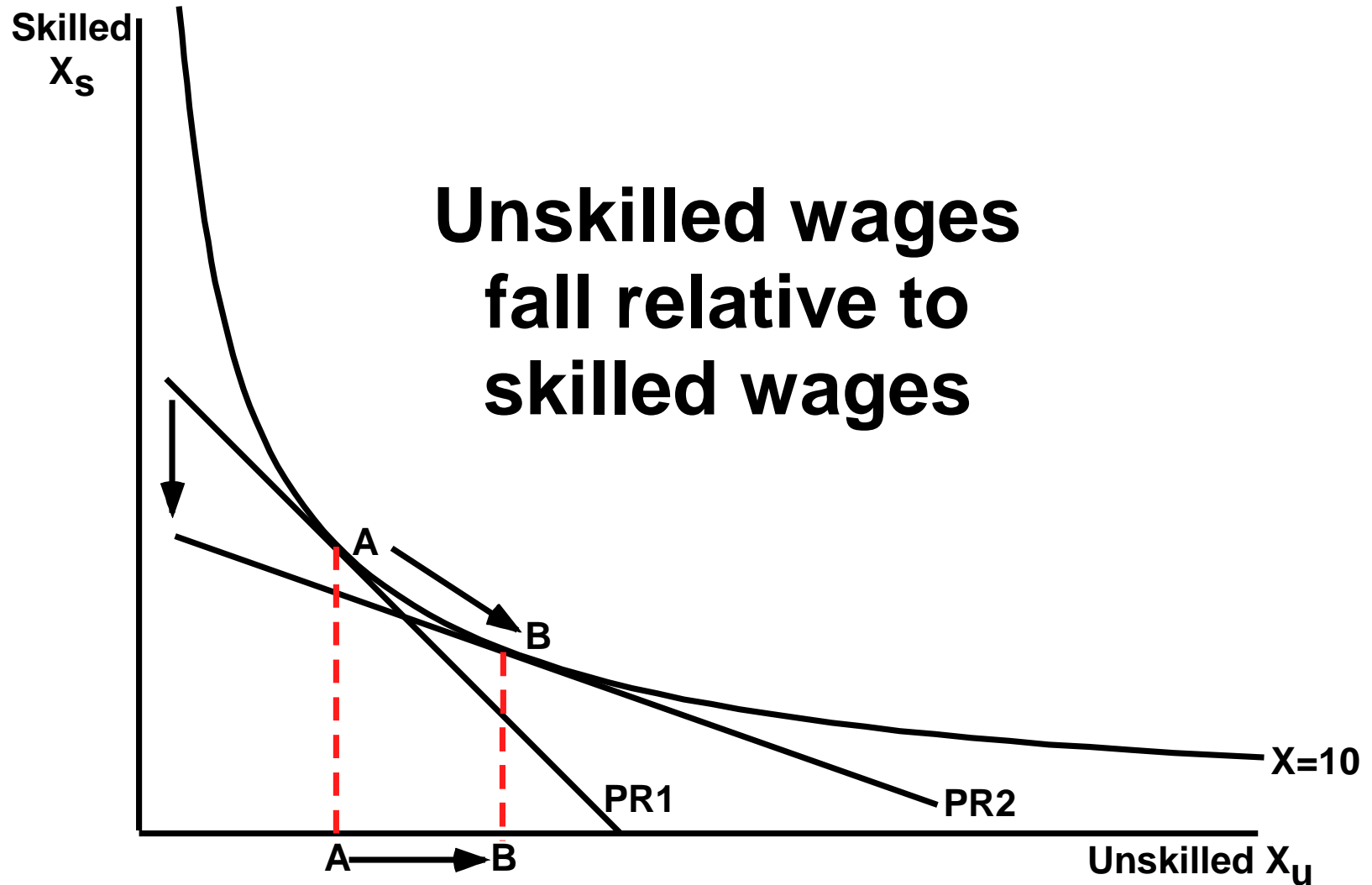
Example 2: Substitution between domestic and imported sources

KEY

Functional Form (diamond)

Inputs or Outputs (rounded box)

Good 1 — up to — Good C

CES

CES
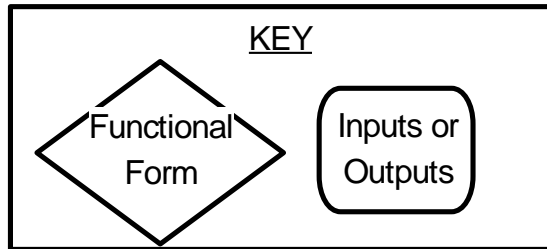
Armington hypothesis:

CES aggregation for different sources

Good-specific parameters

Same structure for all users

Domestic Good 1

Imported Good 1

Domestic Good C

Imported Good C
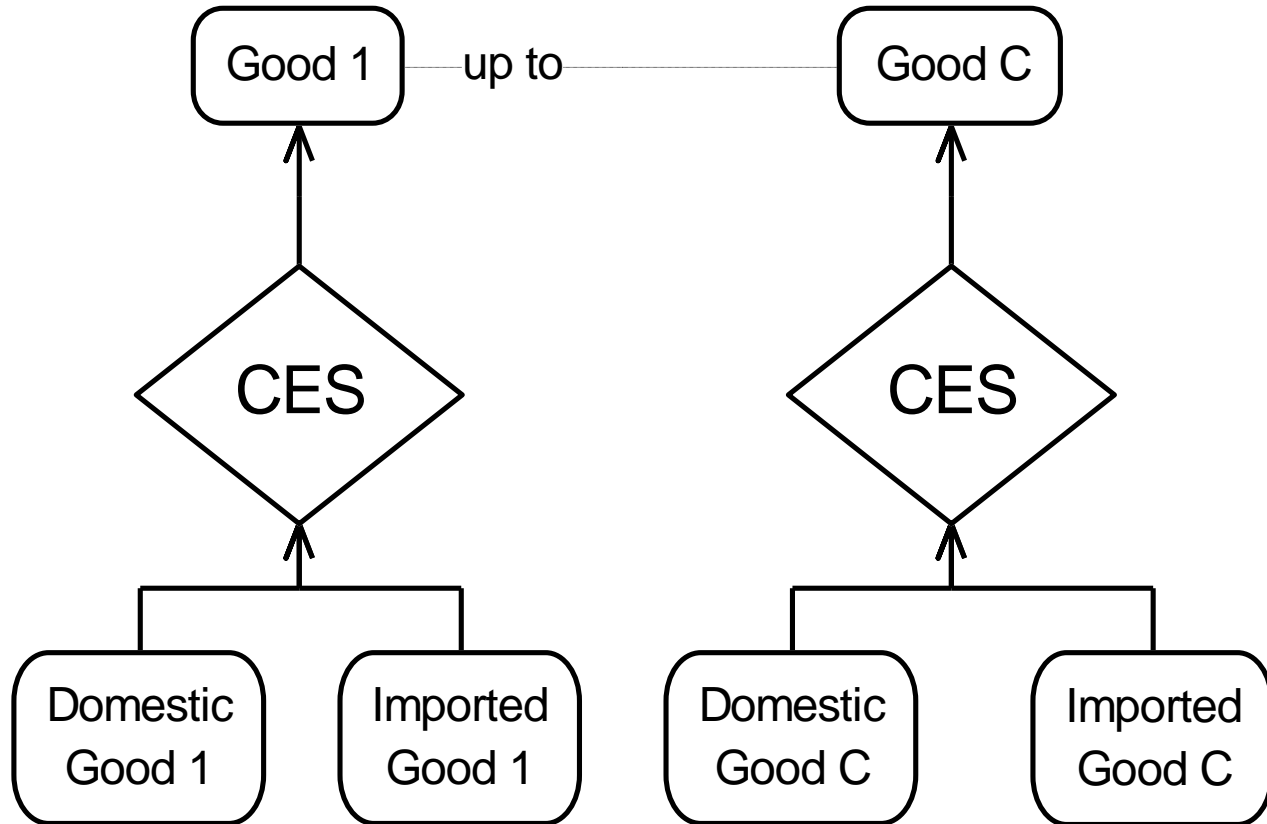
Numerical example

$p = S_d p_d + S_m p_m$ average price of manufactures dom. e imp.

$x_d = x - \sigma(p_d - p)$ demand for domestic manufactures

$x_m = x - \sigma(p_m - p)$ demand for imported manufactures

Let $p_m = -10\%$; $x = p_d = 0$; $S_m = 0.3$ and $\sigma = 2$

Then, <u>cheaper imports</u> imply:

$p = -0.3(-10\%) = -3\%$ *decrease* *in the average price of imp. mnf.*

$x_d = -2[0-(-3\%)] = -6\%$ *decrease* *in domestic demand*

$x_m = -2[-10\%-(-3\%)] = 14\%$ *increase* *in import volume*

Effects on domestic sales are proportional to S_m e σ

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Johansen models

Class of general equilibrium models in which a equilibrium is a vector V, of length n satisfying a system of equations

$$F(V) = 0 \qquad\qquad (1)$$

where F is a vector function of length m.

We assume that F is differentiable and that the number of variables, n, exceeds the number of equations m.

Johansen's approach is to derive from (1) a system of linear equations in which the variables are changes, percentage changes or changes in the logarithms of the components of V.

Illustrative computations

We will assume that the system (1) consists of 2 equations and 3 variables and has the form

$$\begin{cases} V_1^2 V_3 - 1 = 0 \\ V_1 + V_2 - 2 = 0 \end{cases}$$

V_1 and V_2 (endogenous); V_3 (exogenous)

$$\begin{cases} V_1 = V_3^{-1/2} \\ V_2 = 2 - V_3^{-1/2} \end{cases}$$

Illustrative computations (cont.)

Initial solution: $V^I = (V_1^I, V_2^I, V_3^I) = (1,1,1)$

What is the effects on V_1 and V_2 of a shift in V_3 from 1 to 1.1?

$$V_1 = 0.9535$$
$$V_2 = 1.0465$$

Johansen approach:
- Complexity and size of the system (1) normally rule out the possibility of deriving from it explicitly solution equations
- Solve a linearized version of (1)

Step by step

Derive from (1) a differential form

$$A(V)v = 0 \qquad\qquad (2)$$

v is usually interpreted as showing percentage changes or changes in the logarithms of the variables *V*.

Johansen-style computations make use of a initial solution, V^I, with results being reported usually as percentage deviations from this initial solution.

$$A(V) \rightarrow A(V^I) \quad \text{fixed matrix ("model")}$$

Step by step (cont.)

Derivation of (2) is by total differentiation of (1)

$$\begin{bmatrix} 2V_1V_3 & 0 & V_1^2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} dV_1 \\ dV_2 \\ dV_3 \end{bmatrix} = 0$$

Se $V = V^I$,

$$A(V^I)v = 0 \tag{3}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} dV_1 \\ dV_2 \\ dV_3 \end{bmatrix} = 0$$

Step by step (cont.)

On choosing variable 3 to be exogenous:

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} dV_3 = 0$$

$$A_\alpha (V^I) v_\alpha + A_\beta (V^I) v_\beta = 0 \tag{4}$$

$$\begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} = -\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} dV_3$$

$$v_\alpha = -A_\alpha^{-1}(V^I) A_\beta (V^I) v_\beta \tag{5}$$

$$\begin{bmatrix} dV_1 \\ dV_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} dV_3$$

$$v_\alpha = B(V^I) v_\beta \tag{6}$$

Any difference?

<u>Before</u>: V_3↑ 10% → V_1↓ 4.65%, V_2↑ 4.65%

<u>Now</u>: V_3↑ 10% → V_1↓ 5%, V_2↑ 5%

Differences are due to linearization errors…

The operations give us the values of the derivatives or elasticities only for the initial values, V^I, of the variables.

When we move away from V^I, the derivatives or elasticities will change.

In percentage change form:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -\begin{bmatrix} 2 & 0 \\ 0.5 & 0.5 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} v_3$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} v_3$$

Linearization error
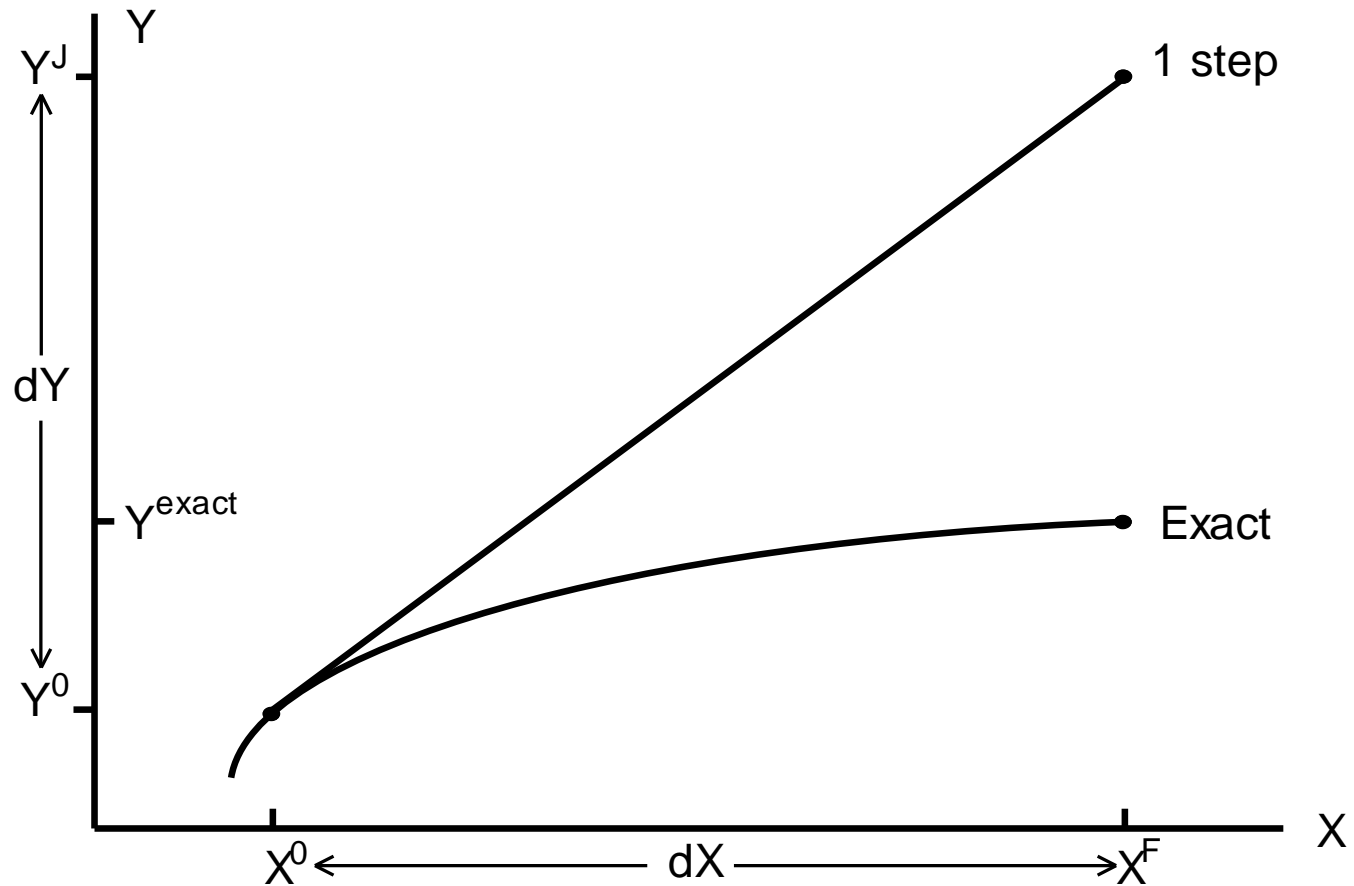
Y^J is *Johansen* estimate
Error is proportionately less for smaller changes

Breaking large changes in X into a number of steps
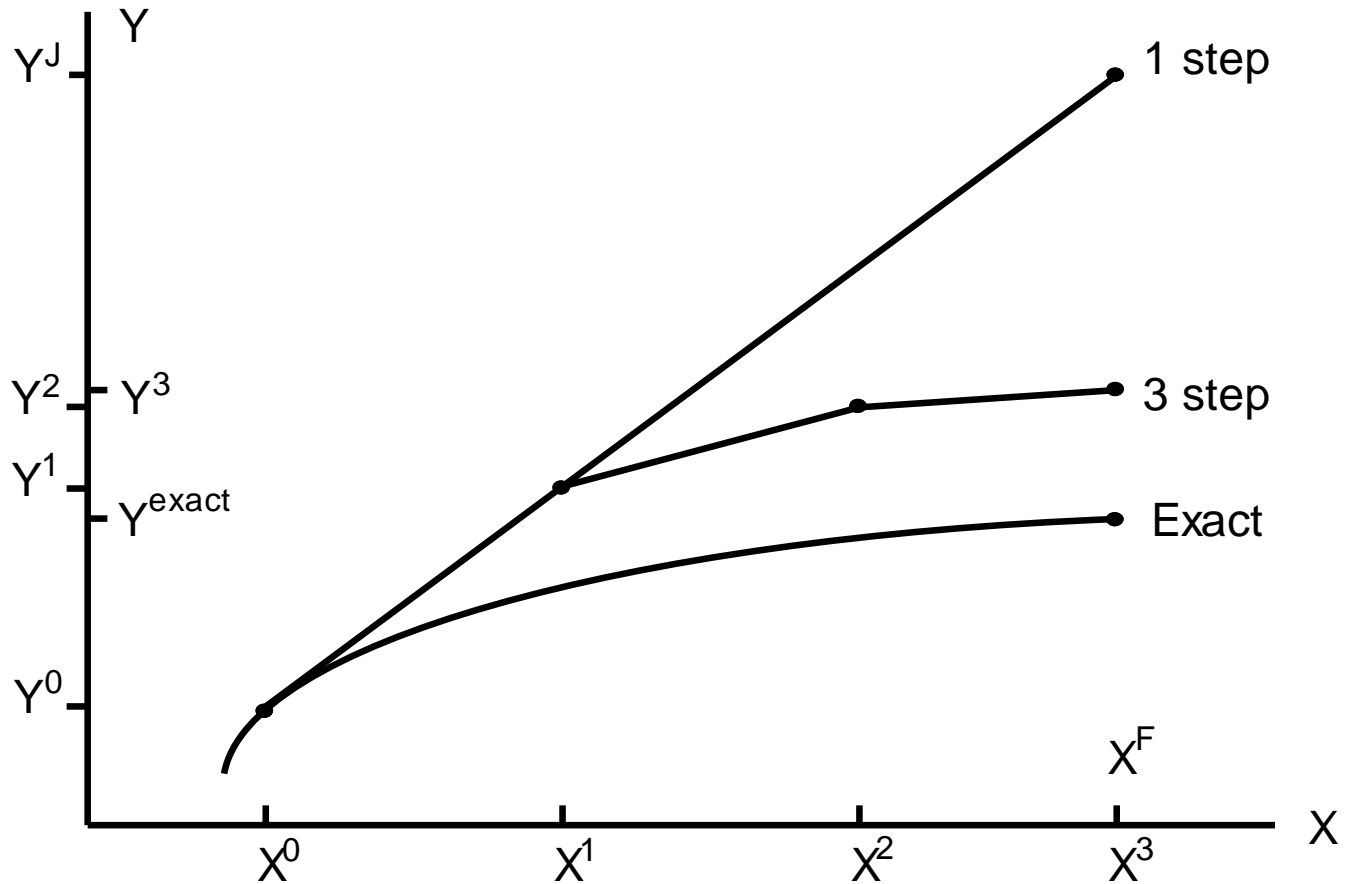
Multi-step process to reduce linearization error

Extrapolating from Johansen and Euler approximations

Method	y	Error
Johansen (1-step)	150%	50%
Euler 2-step	125%	25%
Euler 4-step	112.3%	12.3%
Euler ∞-step (exact)	100%	0

The error follows a rule (*arbitrary* polynomial approximation of continuous function)

Use results from 3 approximate solutions to estimate exact solution + error bound

Summary

1. We start with the model's equations represented in their levels form

2. The equations are linearized: take total differential of each equation

3. Total differential expressions converted to (mostly) % change form

4. Linear equations evaluated at initial solution to the levels model

5. Exogenous variables chosen; model then solved for movements in endogenous variables, given user-specified values for exogenous variables

But, a problem: *linearization error*

Multi-step, extrapolation

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Interregional CGE Models

Implementation

1. Development of a theoretical structure

2. Linearization of the model equations

3. Use of input-output data to provide estimates for the relevant cost and sales shares

4. Development of a flexible computer program to manipulate the linear system

Structure of the model

Two good/sectors (1 and 2)

Two primary factors (3 and 4)

One final user – household (0)

Assumptions

Household sector (0)

$$\max_{X_{i0}} \ U = X_{10}^{\alpha_{10}} X_{20}^{\alpha_{20}} \quad \text{subject to} \quad P_1 X_{10} + P_2 X_{20} = Y$$

Producers (j = 1,2)

$$\min_{X_{ij}} \ C_j = \sum_{i=1}^{4} P_i X_{ij} \quad \text{subject to} \quad X_j = A_j X_{1j}^{\alpha_{1j}} X_{2j}^{\alpha_{2j}} X_{3j}^{\alpha_{3j}} X_{4j}^{\alpha_{4j}}$$

$$A_j > 0, \ \sum_{i=1}^{4} \alpha_{ij} = 1$$

Assumptions (cont.)

Value of output equals the value of inputs

$$C_j = P_j X_j = \sum_{i=1}^{4} X_{ij} P_i, \quad j = 1,2$$

Equilibrium conditions

$$\sum_{j=0}^{2} X_{ij} = X_i, \quad i = 1,2$$

$$\sum_{j=1}^{2} X_{ij} = X_i, \quad i = 3,4$$

Household budget equals factor income

Structural form

Share of input i in total costs of industry j

$$X_{i0} = \frac{\alpha_{i0} Y}{P_i}, \quad i = 1,2 \tag{1}$$

$$X_{ij} = \frac{\left(\alpha_{ij} Q_j X_j \prod_{t=1}^{4} P_t^{\alpha_{tj}} \right)}{P_i}, \quad i = 1, \cdots, 4 \quad j = 1,2 \tag{2}$$

$$P_j = Q_j \prod P_t^{\alpha_{tj}}, \quad j = 1,2 \tag{3}$$

Structural form (cont.)

$$\sum_{j=0}^{2} X_{ij} = X_i, \quad i = 1,2 \tag{4}$$

$$\sum_{j=1}^{2} X_{ij} = X_i, \quad i = 3,4 \tag{5}$$

$$P_1 = 1 \tag{6}$$

Variables:

$$X_{i0} \quad i = 1,2$$

$$X_{ij} \quad i = 1, \cdots, 4 \;\; j = 1,2$$

$$P_i \quad i = 1, \cdots, 4$$

$$X_i = 1, \cdots, 4$$

$$Y$$

Percent-change equations – examples

Levels form: $A = B + C$

Ordinary
change form: $\Delta A = \Delta B + \Delta C$

Convert to % $A(100.\Delta A/A) = B(100.\Delta B/B) + C(100.\Delta C/C)$
change form: $A\ \underline{a} \qquad\quad = B\ \underline{b} \qquad\quad + C\ \underline{c}$

Typically two ways of expressing % change form

Intermediate form: $A\,a = B\,b + C\,c$

Percentage change (share) form: $a = S_b\,b + S_c\,c$
                                           where $S_b = B/A$; $S_c = C/A$

Percent-change equations – examples

Levels form: $A = B\,C$

Ordinary
change form: $\Delta A = \Delta B\,C + \Delta C\,B$

Convert to % $A(100.\Delta A/A) = BC(100.\Delta B/B) + BC(100.\Delta C/C)$
change form: $A\quad a \qquad\quad = BC\quad b \qquad + BC\quad c$
                      $a \qquad\quad = \qquad b \qquad + \qquad c$

PRACTICE: $X = F\,P^{\varepsilon}$

- Ordinary change and percent change are both linearized
- Linearized equations easier for computers to solve
- % change equations easier for economists to understand: elasticities

Percent-change numerical example

Levels form $Z = X*Y$

Ordinary Change form $\Delta Z = Y*\Delta X + X*\Delta Y$ [+ $\Delta X \, \Delta Y$]

2nd-order

Multiply by 100: $100*\Delta Z = 100*Y*\Delta X + 100*X*\Delta Y$

Define x = % change in X, so $X*x = 100\Delta X$

$Z*z = X*Y*x + X*Y*y$

Divide by $Z=X*Y$ to get:

Percent Change form $z = x + y$

Initially   X=4, Y=5, so Z = X*Y = 20
Suppose x=25%, y=20% [ie, X:4$\Rightarrow$5, Y:5$\Rightarrow$6]
Linear approximation z = x + y gives z = 45%
True answer: 30 = 5*6… = 50% more than original 20
Error 5% is 2nd order term: z = x+y + x*y/100

Note: reduce shocks by a factor of 10, error by factor of 100

Linearized version

$$x_{i0} = y - p_i \quad i = 1,2$$

$$x_{ij} = x_j - \left(p_i - \sum_{t=1}^{4} \alpha_{tj} p_t \right) \quad i = 1, \cdots, 4 \quad j = 1,2$$

$$p_j = \sum_{t=1}^{4} \alpha_{tj} p_t \quad j = 1,2$$

Variables:

$$\sum_{j=0}^{2} x_{ij} \beta_{ij} = x_i \quad i = 1,2$$

$$\sum_{j=1}^{2} x_{ij} \beta_{ij} = x_i \quad i = 3,4$$

$$p_1 = 0$$

$$
\begin{array}{ll}
x_{i0} & i = 1,2 \\
x_{ij} & i = 1,\cdots,4 \quad j = 1,2 \\
p_i & i = 1,\cdots,4 \\
x_i & = 1,\cdots,4 \\
y &
\end{array}
$$

$$\beta_{ij} = X_{ij} / X_i, \quad i = 1,...,4 \quad j = 0,1,2$$

Initial solution

Input-output data

Convention: one unit of good or factor *i* is the amount which costs $1 in the base period

Parameters of the structural form: $\alpha_{i0}, \alpha_{ij}, Q_j$

Calibration guarantees that structural form equations are satisfied by the suggested values for the parameters

Matrix $A(V^I)$

Matrix $A(V^I)$ can be evaluated using the input-output data

Equivalent to evaluate the coefficients of the model!

Exercise: representation of the matrix $A(V^I)$ of the linearized system

Linearized version

$$x_{i0} = y - p_i \quad i = 1,2$$

$$x_{ij} = x_j - \left(p_i - \sum_{t=1}^{4} \alpha_{tj} p_t \right) \quad i = 1, \cdots, 4 \quad j = 1,2$$

$$p_j = \sum_{t=1}^{4} \alpha_{tj} p_t \quad j = 1,2$$

Variables:

$$\sum_{j=0}^{2} x_{ij} \beta_{ij} = x_i \quad i = 1,2$$

$$\sum_{j=1}^{2} x_{ij} \beta_{ij} = x_i \quad i = 3,4$$

$$p_1 = 0$$

$$\begin{array}{ll} x_{i0} & i = 1,2 \\ x_{ij} & i = 1,\cdots,4 \quad j = 1,2 \\ p_i & i = 1,\cdots,4 \\ x_i & = 1,\cdots,4 \\ y & \end{array}$$

$$\beta_{ij} = X_{ij} / X_i, \quad i = 1,\dots,4 \quad j = 0,1,2$$

Input-output data

Table E3.3.1
Input-Output Data (Flows in dollars)

		Industry		Households	Total Sales
		1	2		
Commodity	1	4	2	2	8
	2	2	6	4	12
Primary Factors	Labor 3	1	3		4
	Capital 4	1	1		2
Production		8	12	6	

Table E3.4.2

*Answer to Exercise 3.4: The Transpose of the Matrix A(V^I) for the Stylized Johansen Model**

19 rows

	(E3.2.1)		(E3.2.2)								(E3.2.3)		(E3.2.4)		(E3.2.5)		(E3.2.6)
y	-1	-1															
x_{10}	1												-.25				
x_{20}		1												-.3			
x_{11}			1										-.5				
x_{21}				1										-.16			
x_{31}					1										-.25		
x_{41}						1										-.5	
x_{12}							1						-.25				
x_{22}								1						-.5			
x_{32}									1						-.75		
x_{42}										1						-.5	
x_1			-1	-1	-1	-1					1						
x_2							-1	-1	-1	-1			1				
x_3															1		
x_4																1	
p_1	1		.5	-.5	-.5	-.5	.83	-.16	-.16	-.16	.5	-.16					1
p_2		1	-.25	.75	-.25	-.25	-.5	.5	-.5	-.5	-.25	.5					
p_3			-.125	-.125	.875	-.125	-.25	-.25	.75	-.25	-.125	-.25					
p_4			-.125	-.125	-.125	.875	-.083	-.083	-.083	.916	-.125	-.083					

* For typographical convenience we have listed the columns of A(V^I) as rows. Numbers of the form .83, .16, etc. are to be read as .8333..., .1666..., etc.

The model

A(VI)

	y	x10	x20	x11	x21	x31	x41	x12	x22	x32	x42	x1	x2	x3	x4	p1	p2	p3	p4
	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0.5	-0.25	-0.125	-0.125
	0	0	0	0	1	0	0	0	0	0	0	-1	0	0	0	-0.5	0.75	-0.125	-0.125
	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	-0.5	-0.25	0.875	-0.125
	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	-0.5	-0.25	-0.125	0.875
	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0.833	-0.5	-0.25	-0.0833
	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	-0.166	0.5	-0.25	-0.0833
	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	-0.166	-0.5	0.75	-0.0833
	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	-0.166	-0.5	-0.25	0.9166
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	-0.25	-0.125	-0.125
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.166	0.5	-0.25	-0.0833
	0	-0.25	0	-0.5	0	0	0	-0.25	0	0	0	1	0	0	0	0	0	0	0
	0	0	-0.333	0	-0.166	0	0	0	-0.5	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	-0.25	0	0	0	-0.75	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	-0.5	0	0	0	-0.5	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Aalfa

	y	x10	x20	x11	x21	x31	x41	x12	x22	x32	x42	x1	x2	p1	p2	p3	p4	Abeta	x3	x4
	-1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		0	0
	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0		0	0
	0	0	0	1	0	0	0	0	0	0	0	-1	0	0.5	-0.25	-0.125	-0.125		0	0
	0	0	0	0	1	0	0	0	0	0	0	-1	0	-0.5	0.75	-0.125	-0.125		0	0
	0	0	0	0	0	1	0	0	0	0	0	-1	0	-0.5	-0.25	0.875	-0.125		0	0
	0	0	0	0	0	0	1	0	0	0	0	-1	0	-0.5	-0.25	-0.125	0.875		0	0
	0	0	0	0	0	0	0	1	0	0	0	0	-1	0.833	-0.5	-0.25	-0.0833		0	0
	0	0	0	0	0	0	0	0	1	0	0	0	-1	-0.166	0.5	-0.25	-0.0833		0	0
	0	0	0	0	0	0	0	0	0	1	0	0	-1	-0.166	-0.5	0.75	-0.0833		0	0
	0	0	0	0	0	0	0	0	0	0	1	0	-1	-0.166	-0.5	-0.25	0.9166		0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	-0.25	-0.125	-0.125		0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.166	0.5	-0.25	-0.0833		0	0
	0	-0.25	0	-0.5	0	0	0	-0.25	0	0	0	1	0	0	0	0	0		0	0
	0	0	-0.333	0	-0.166	0	0	0	-0.5	0	0	0	1	0	0	0	0		0	0
	0	0	0	0	0	-0.25	0	0	0	-0.75	0	0	0	0	0	0	0		1	0
	0	0	0	0	0	0	-0.5	0	0	0	-0.5	0	0	0	0	0	0		0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		0	0

(-)invAalfa

0.340	0.660	0.680	0.329	0.150	0.200	0.340	0.991	0.451	0.200	1.042	-0.778	1.359	1.982	0.601	0.401	-1.000
-0.660	0.660	0.680	0.329	0.150	0.200	0.340	0.991	0.451	0.200	1.042	-0.778	1.359	1.982	0.601	0.401	0.000
0.330	-0.330	0.660	0.334	0.175	0.150	0.330	1.006	0.526	0.150	-0.518	0.392	1.319	2.012	0.701	0.301	-0.001
-0.260	0.260	-1.520	0.130	0.150	0.200	-0.260	0.390	0.450	0.200	3.441	0.421	-1.040	0.781	0.601	0.400	-0.001
-0.270	0.270	-0.540	-0.865	0.175	0.150	-0.270	0.405	0.525	0.150	1.881	1.591	-1.080	0.811	0.701	0.300	-0.002
-0.300	0.300	-0.600	0.150	-0.750	0.000	-0.300	0.451	0.750	0.000	1.201	-0.899	-1.200	0.901	1.001	0.000	0.000
-0.200	0.200	-0.400	0.100	0.000	-0.500	-0.200	0.300	0.000	0.500	0.801	-0.599	-0.800	0.601	0.000	1.000	0.000
0.140	-0.140	0.280	-0.070	0.150	0.200	-0.860	-0.210	0.450	0.200	1.840	1.620	0.560	-0.421	0.600	0.400	-0.002
0.130	-0.130	0.260	-0.065	0.175	0.150	0.130	-1.195	0.525	0.150	0.280	2.790	0.520	-0.390	0.700	0.300	-0.002
0.100	-0.100	0.200	-0.050	0.250	0.000	0.100	-0.150	-0.250	0.000	-0.400	0.300	0.400	-0.300	1.000	0.000	0.000
0.200	-0.200	0.400	-0.100	0.000	0.500	0.200	-0.300	0.000	-0.500	-0.801	0.599	0.800	-0.601	0.000	1.000	0.000
-0.260	0.260	-0.520	0.130	0.150	0.200	-0.260	0.390	0.450	0.200	2.441	0.421	-1.040	0.781	0.601	0.400	-0.001
0.130	-0.130	0.260	-0.065	0.175	0.150	0.130	-0.195	0.525	0.150	0.280	1.790	0.520	-0.390	0.700	0.300	-0.002
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000
0.010	-0.010	0.020	-0.005	-0.025	0.050	0.010	-0.015	-0.075	0.050	1.560	-1.170	0.040	-0.030	-0.100	0.100	-0.999
0.040	-0.040	0.080	-0.020	-0.100	0.200	0.040	-0.060	-0.300	0.200	2.240	1.320	0.160	-0.120	-0.400	0.400	-1.001
-0.060	0.060	-0.120	0.030	0.150	-0.300	-0.060	0.090	0.450	-0.300	2.640	1.020	-0.240	0.180	0.600	-0.600	-1.001

B(VI)

0.60	0.40	y
0.60	0.40	x10
0.70	0.30	x20
0.60	0.40	x11
0.70	0.30	x21
1.00	0.00	x31
0.00	1.00	x41
0.60	0.40	x12
0.70	0.30	x22
1.00	0.00	x32
0.00	1.00	x42
0.60	0.40	x1
0.70	0.30	x2
0.00	0.00	p1
-0.10	0.10	p2
-0.40	0.40	p3
0.60	-0.60	p4

Outline

Introduction

Structure of a CGE Model

The Johansen Approach

Stylized Johansen Model

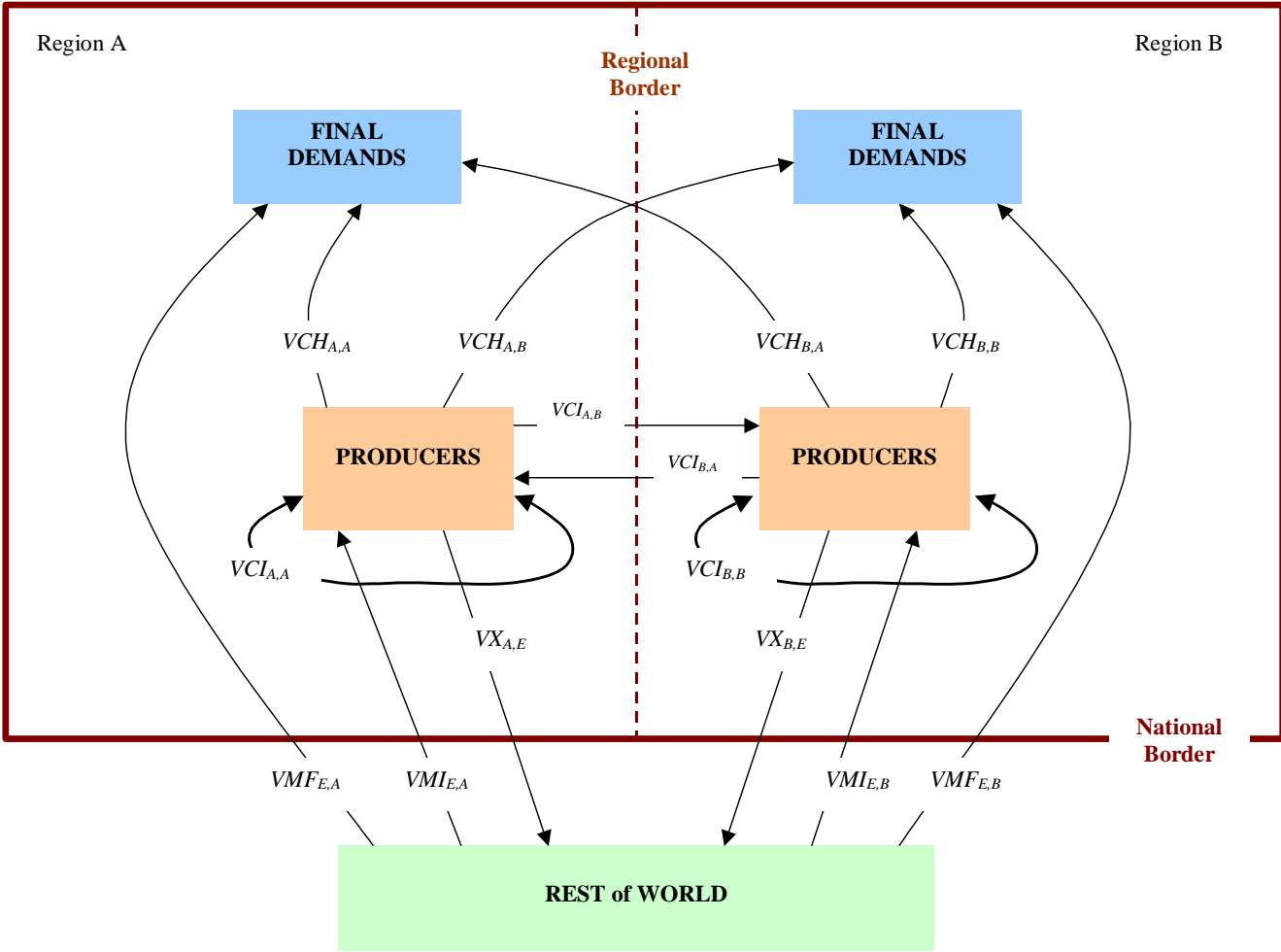✓ Interregional CGE Models

Features of a bottom-up ICGE model

Multi-sectoral, multi-regional bottom-up CGE models

- each region is modeled as an economy in its own right
- region-specific prices
- region-specific industries
- region-specific consumers

Database makes allowance for interregional, intra-regional and international trade

- explicit representations of regional and central government financial accounts
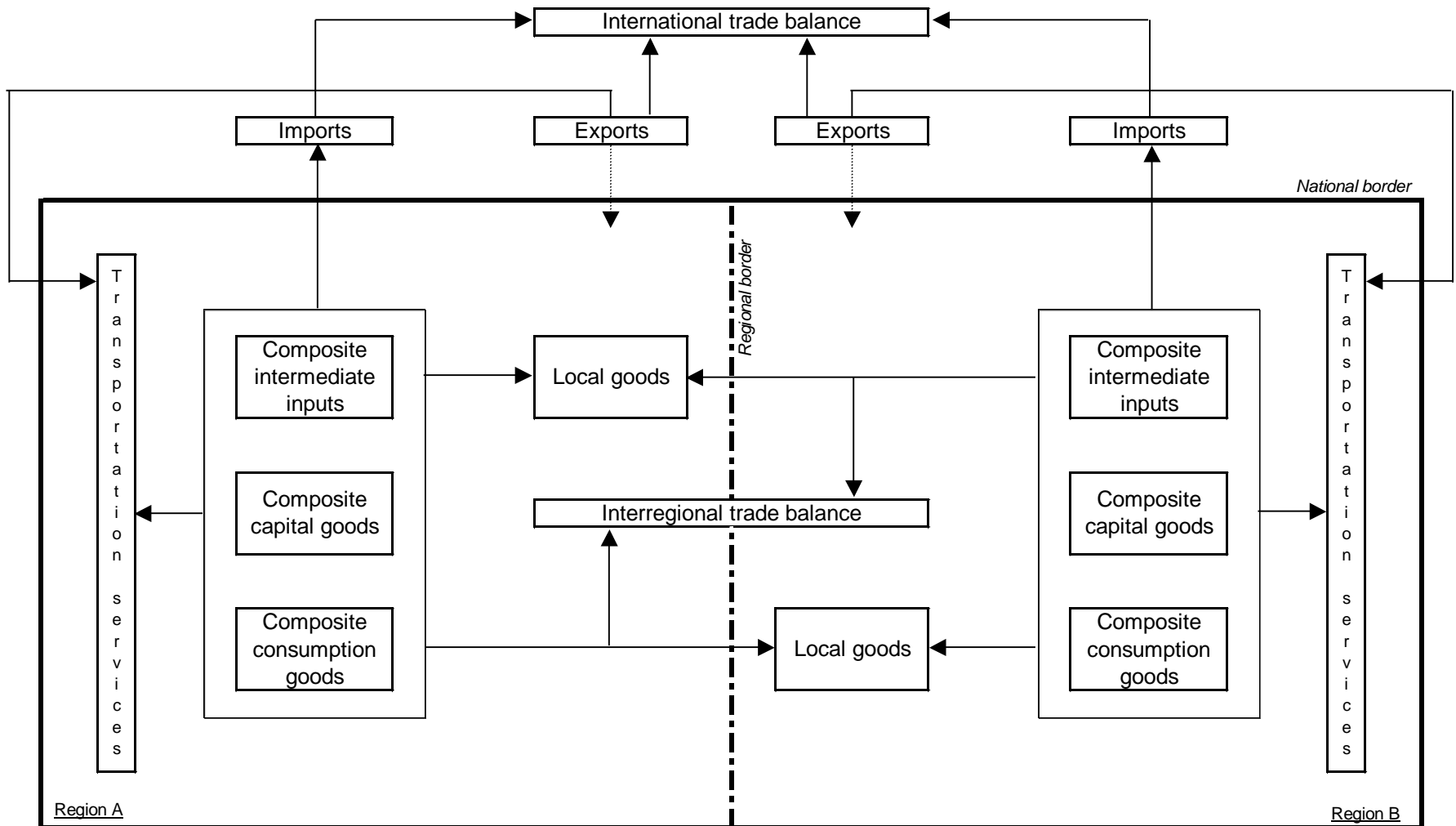
Stylized flows

Region A   **Regional Border**   Region B

FINAL DEMANDS

FINAL DEMANDS

$VCH_{A,A}$ $VCH_{A,B}$ $VCH_{B,A}$ $VCH_{B,B}$

$VCI_{A,B}$

PRODUCERS

$VCI_{B,A}$

PRODUCERS

$VCI_{A,A}$

$VCI_{B,B}$

$VX_{A,E}$ $VX_{B,E}$

National Border

$VMF_{E,A}$ $VMI_{E,A}$ $VMI_{E,B}$ $VMF_{E,B}$

REST of WORLD

Embedded SAM

| | REGION 1 | | | | | | REGION R | | | | | | FEDERAL GOV. | IMPORTS | EXPORTS | ACCUM. | ROW | TOTAL |
	FACTORS	AGENTS H	AGENTS G	SECTORS	LOCAL PRODUCTS	COMPOSITE PRODUCTS (SUPPLY)	FACTORS	AGENTS H	AGENTS G	SECTORS	LOCAL PRODUCTS	COMPOSITE PRODUCTS (SUPPLY)						
REGION 1 — FACTORS				W^1														W^1
REGION 1 — AGENTS H	Wh^{11}		$Thrg^1$				Wh^{1R}						$Thfg^1$					Yh^1
REGION 1 — AGENTS G		$IDrgh^1$			$TIrg^1$								Tfg^1					Yrg^1
REGION 1 — SECTORS					PMI^1										E^1			PCF^1
REGION 1 — LOCAL PRODUCTS						Ppm^{11}						Ppm^{1R}						Ppm^1
REGION 1 — COMPOSITE PRODUCTS (DEMAND)		Ch^1	Crg^1	CI^1									Cfg^1			I^1		D^1
REGION R — FACTORS										W^R								W^R
REGION R — AGENTS H	Wh^{R1}						Wh^{RR}		$Thrg^R$				$Thfg^R$					Yh^R
REGION R — AGENTS G								$IDrgh^R$			$TIrg^R$		Tfg^R					Yrg^R
REGION R — SECTORS											PMI^R				E^R			PCF^1
REGION R — LOCAL PRODUCTS						Ppm^{R1}						Ppm^{RR}						Ppm^R
REGION R — COMPOSITE PRODUCTS (DEMAND)								Ch^R	Crg^R	CI^R			Cfg^R			I^R		D^R
FEDERAL GOVERNMENT		$IDfgh^1$			$TIfg^1$			$IDfgh^R$			$TIfg^R$			Tim	Tex			Yfg
IMPORTS						Ipm^1						Ipm^R						Ipm
EXPORTS																	Epm	Epm
ACCUMULATION		Sh^1	Srg^1					Sh^R	Srg^R				Sfg				SBC	I
ROW														M				Yrw
FINANCIAL/ASSET ADJUSTMENT		FAh^1	$FArg^1$					FAh^R	$FArg^R$				$FAfg$					FA
TOTAL	W^1	Yh^1	Yrg^1	PCF^1	Ppm^1	O^1	W^R	Yh^R	Yrg^R	PCF^R	Ppm^R	O^R	Yfg	Ipm	Epm	I	Yrw	

The role of transportation services

Core database

	Size	ABSORPTION MATRIX					
		1	2	3	4	5	6
		Producers	Investors	Household	Export	Regional Govt.	Federal Govt.
	Size	J x Q	J x Q	Q	1	Q	Q
Basic Flows	I x S	BAS1	BAS2	BAS3	BAS4	BAS5	BAS6
Margins	I x S x R	MAR1	MAR2	MAR3	MAR4	MAR5	MAR6
Taxes	I x S	TAX1	TAX2	TAX3	TAX4	TAX5	TAX6
Labor	1	LABR					
Capital	1	CPTL					
Other	1	OCTS					

I = number of commodities
J = number of industries
R = number of commodities used as margins
Q = number of regions
S = Q domestic regions + 1 foreign import

Features of database

Commodity flows are valued at "basic prices" (**BAS**):
- do not include user-specific taxes or margins

For each user of each imported good and each domestic good, there are numbers showing:
- tax levied on that usage (**TAX**)
- usage of margins – transport (**MAR**)

Single-production:
- each commodity may be produced by one industry
- each industry may produce one commodity
- multi-production may be introduced

For each industry the total cost of production is equal to the total value of output

For each commodity the total value of sales is equal to the total value of output

Features of database (cont.)

Domestic producers
- J industries in Q regions

Investors
- J industries in Q regions

Households
- one representative household for each of the Q regions

Each of the I commodity types can be obtained from the region, from other regions, or imported from overseas

Features of database (cont.)

Aggregate foreign purchaser of exports

Other demand category corresponding to the Q regional governments

Other demand category corresponding to the central government in the Q regions

Commodity taxes and Margins explicitly recognised

Core database

	Size	ABSORPTION MATRIX					
		1	2	3	4	5	6
		Producers	Investors	Household	Export	Regional Govt.	Federal Govt.
	Size	J x Q	J x Q	Q	1	Q	Q
Basic Flows	I x S	BAS1	BAS2	BAS3	BAS4	BAS5	BAS6
Margins	I x S x R	MAR1	MAR2	MAR3	MAR4	MAR5	MAR6
Taxes	I x S	TAX1	TAX2	TAX3	TAX4	TAX5	TAX6
Labor	1	LABR					
Capital	1	CPTL					
Other	1	OCTS					

I = number of commodities
J = number of industries
R = number of commodities used as margins
Q = number of regions
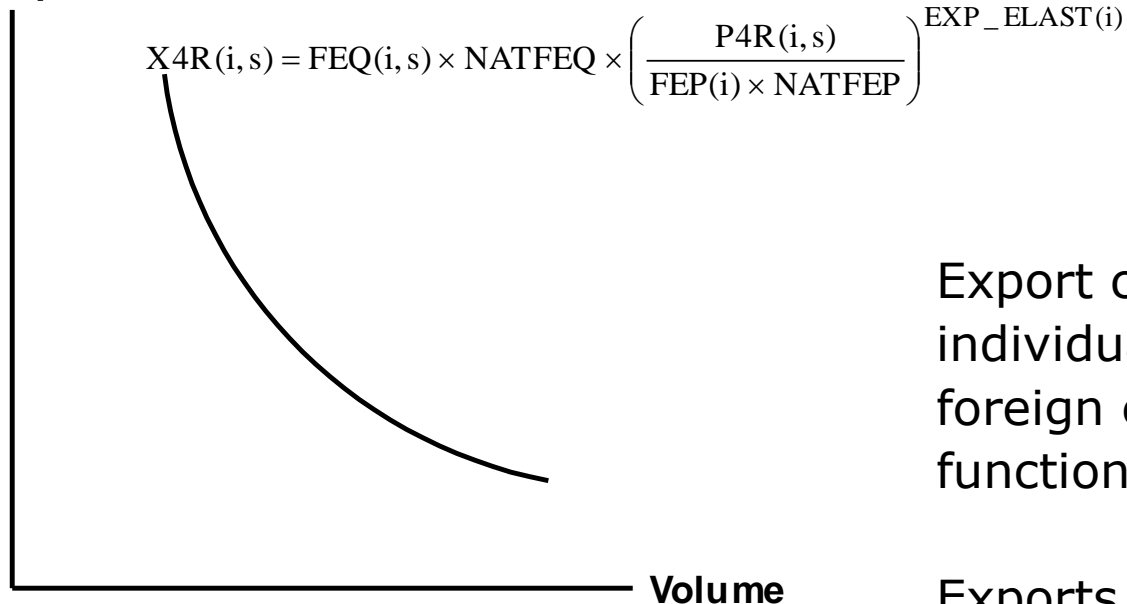*S = Q domestic regions + 1 foreign import*

Production nest
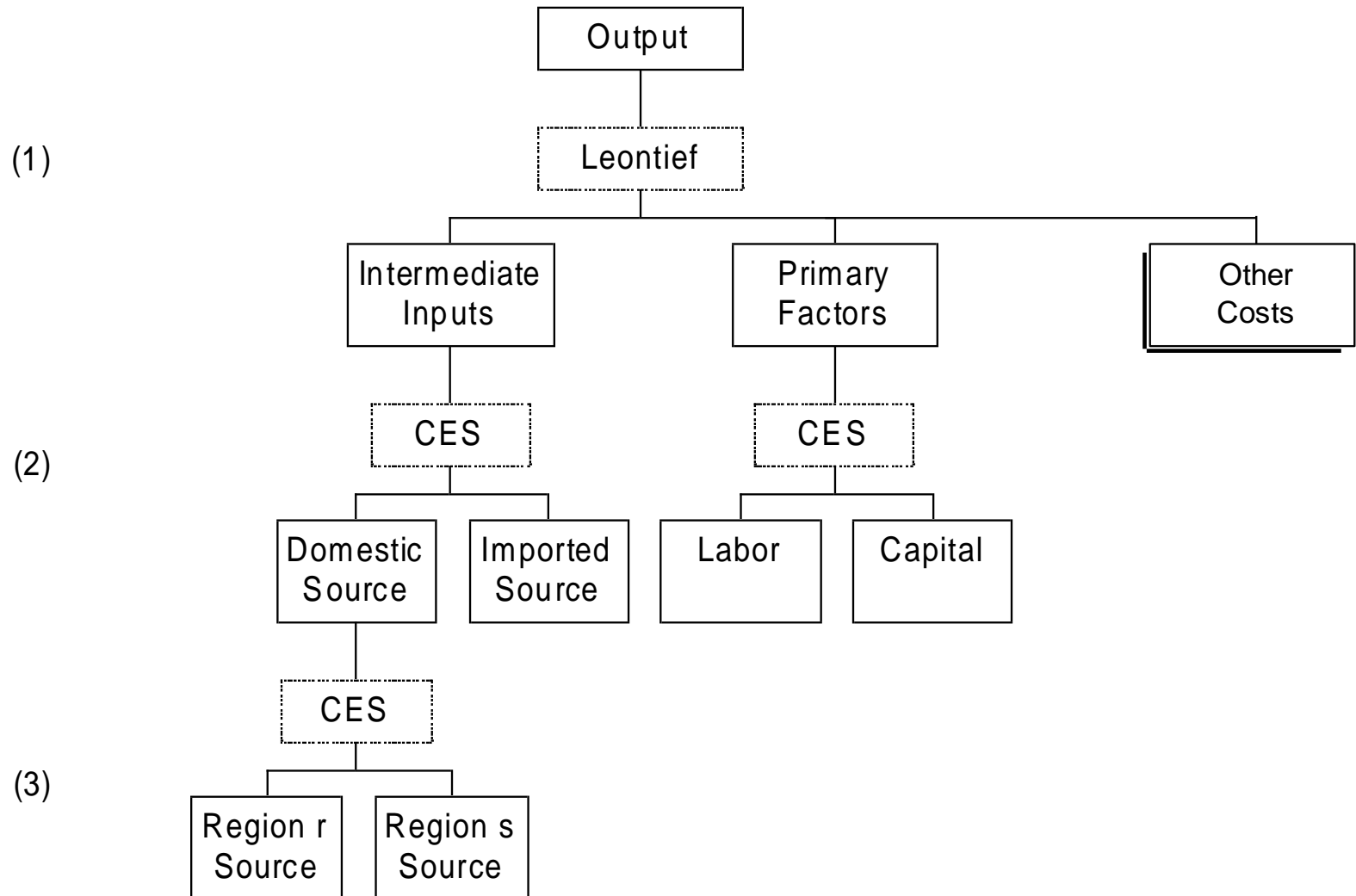
Household demand
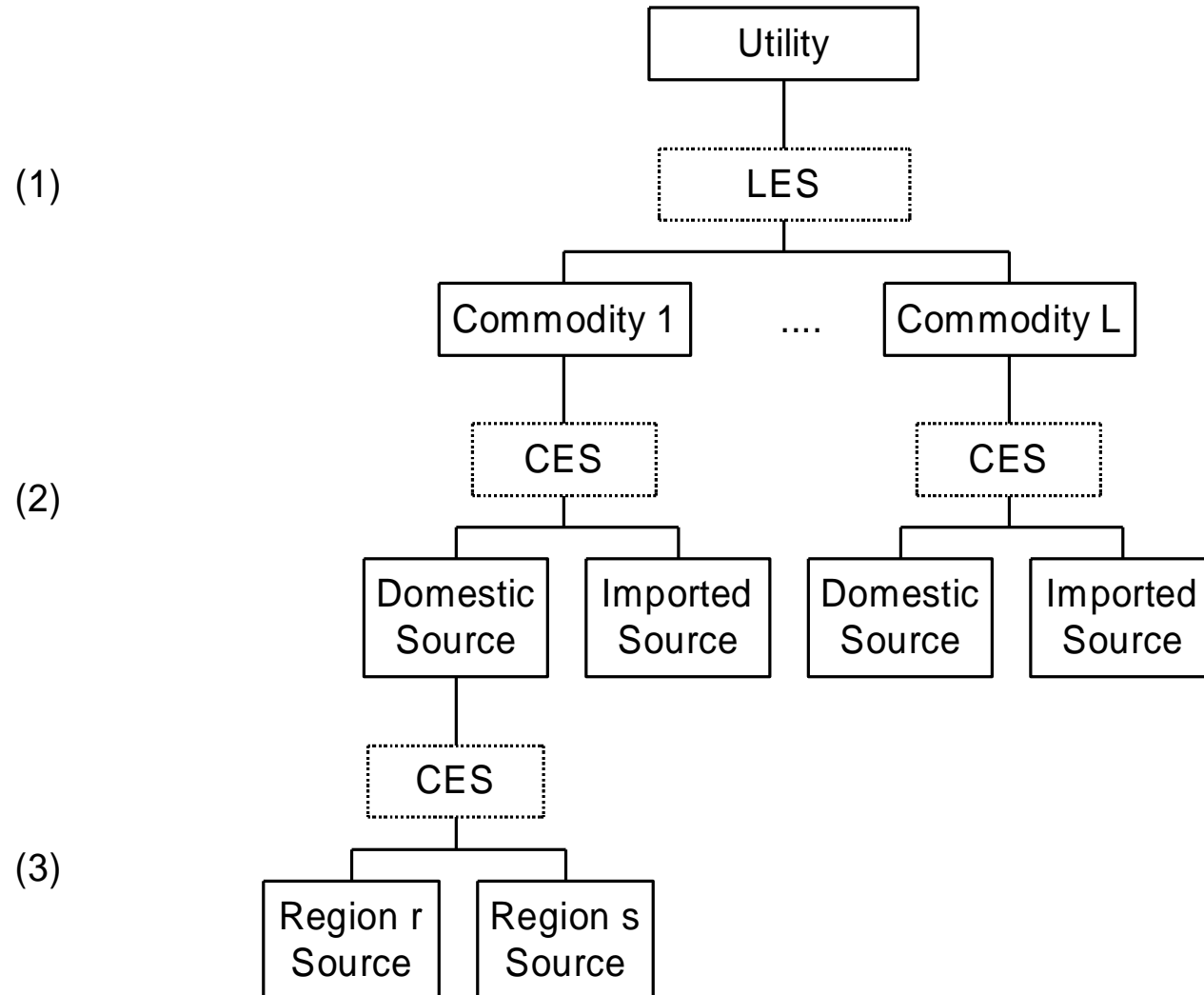
Foreign export demand

Export Price

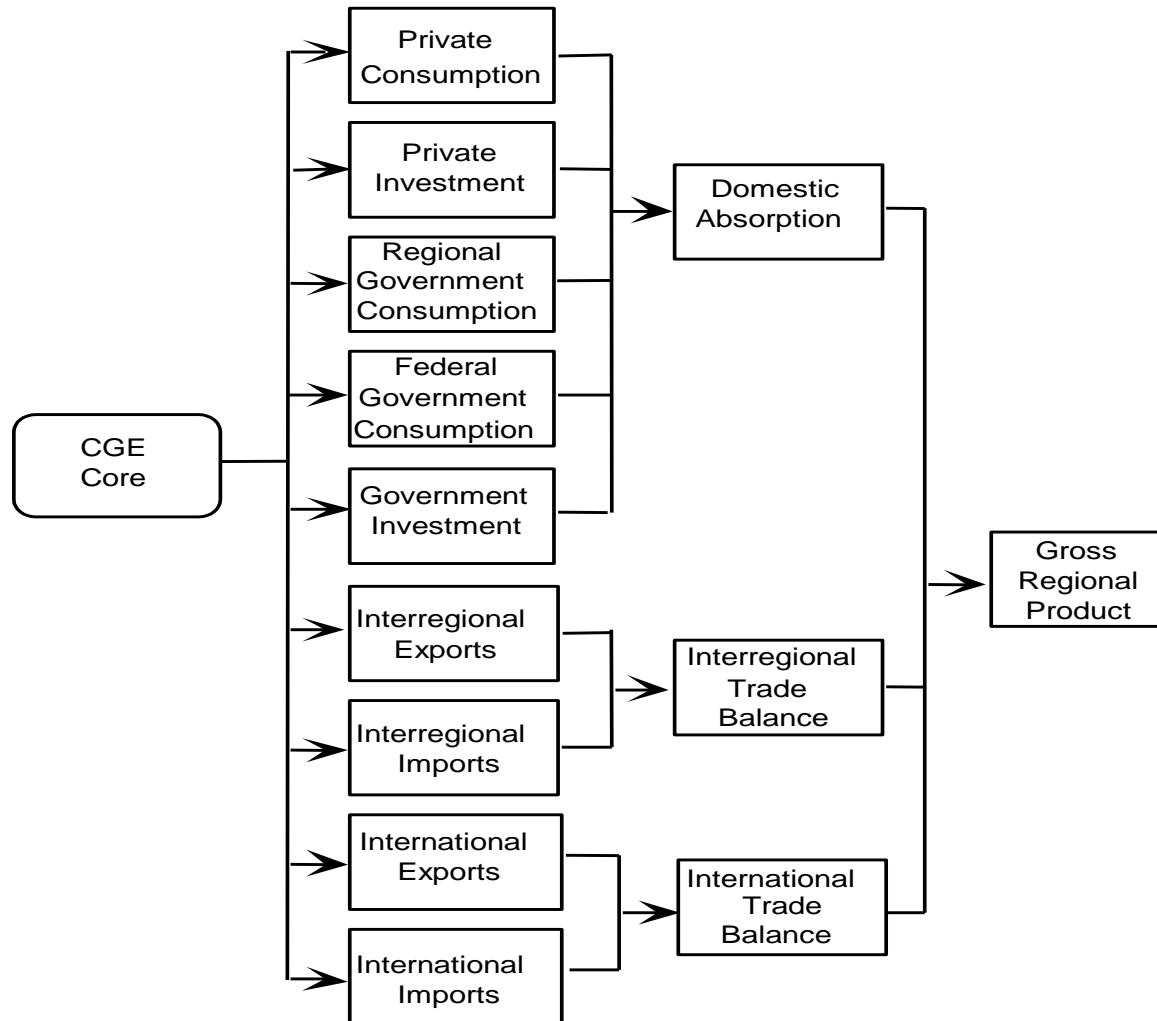$$X4R(i,s) = FEQ(i,s) \times NATFEQ \times \left(\frac{P4R(i,s)}{FEP(i) \times NATFEP} \right)^{EXP\_ELAST(i)}$$
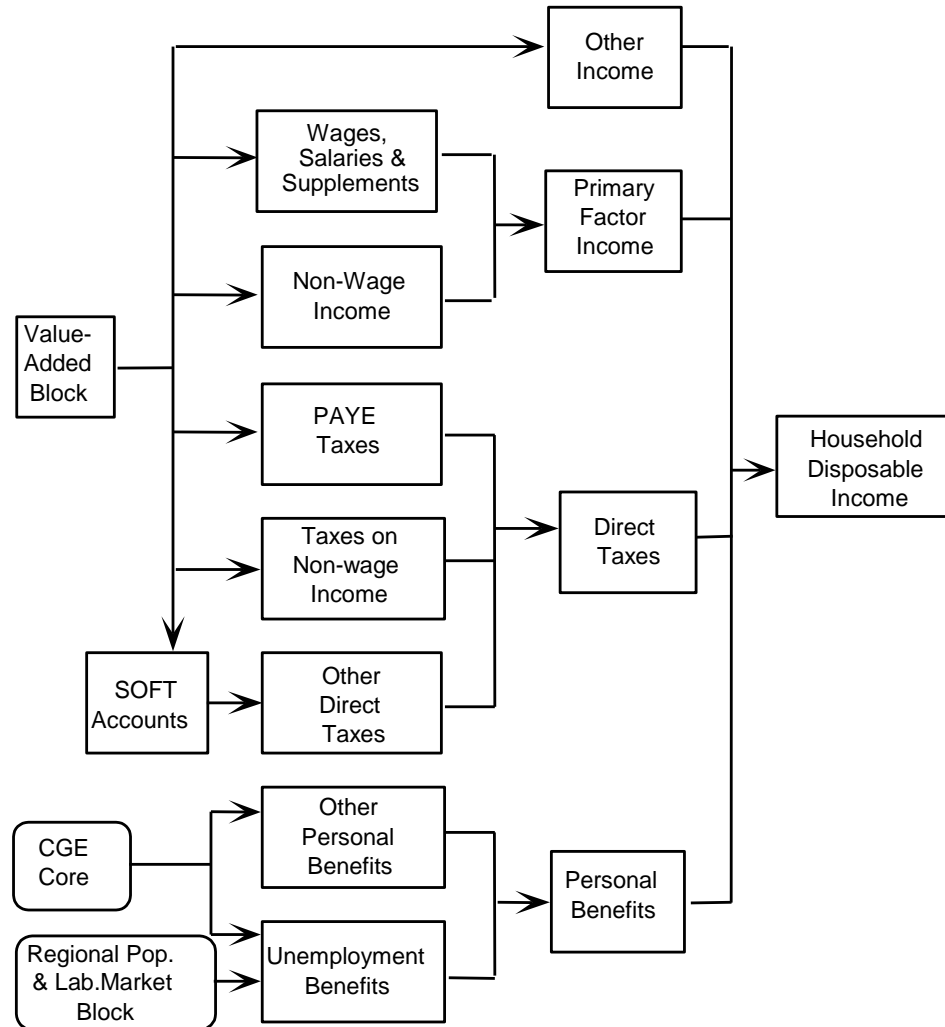
Volume

Export commodities face individual downward-sloping foreign export demand functions

Exports of product i from source s are distinguished from exports of i from source r (r not equal s)
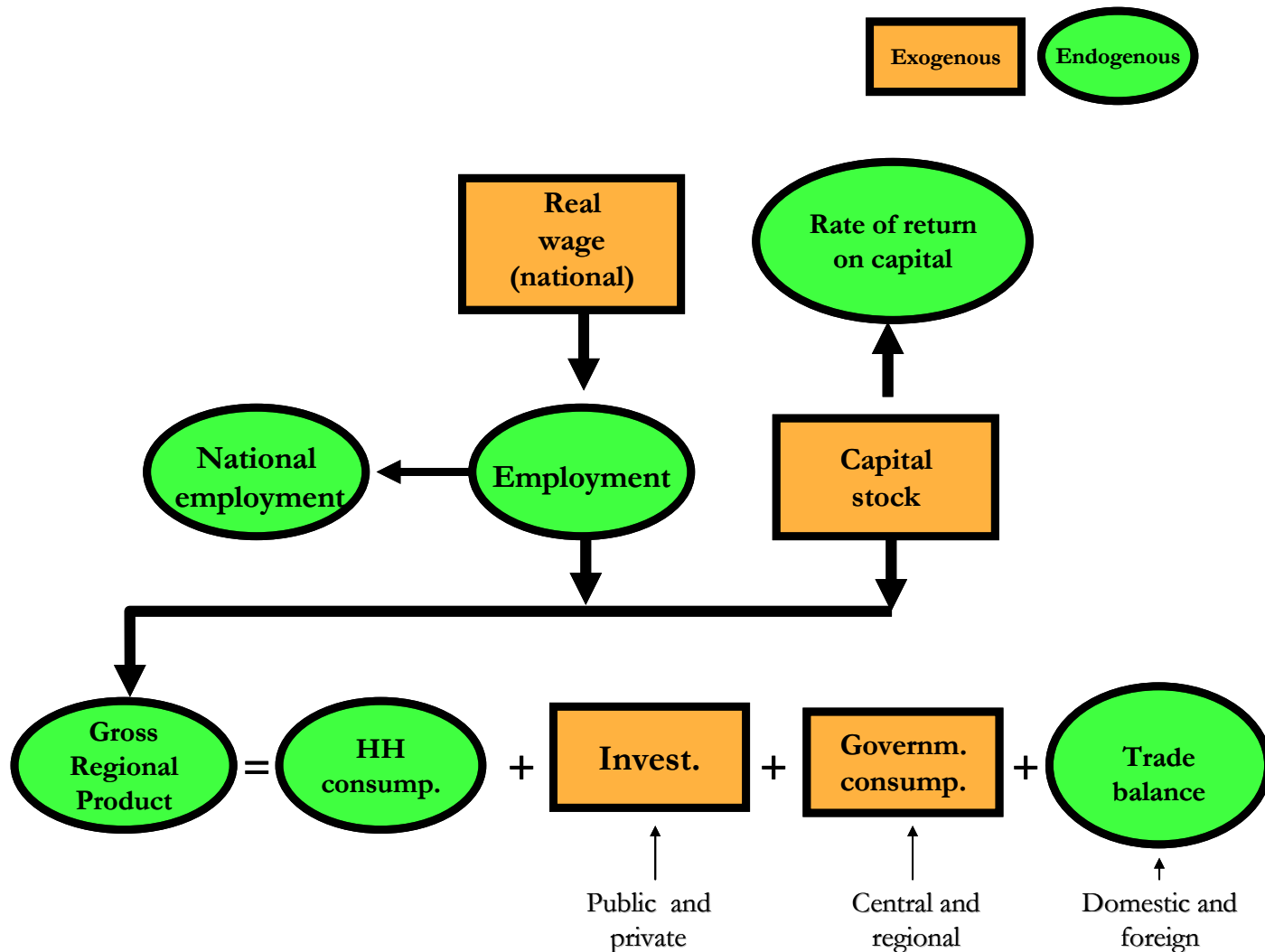
Gross Regional Product

Household disposable income

Possible short-run environment

Possible long-run environment